FINAL STATE INTERACTION IN ELECTRO-DISINTEGRATION OF NUCLEI and REALISTIC WAVE FUNCTIONS

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ABSTRACT

A new linked cluster expansion for the calculation of ground state observables of complex nuclei with realistic interactions has been developed [1]; using the V8′ potential the ground state energy, density and momentum distribution of complex nuclei have been calculated and found to be in good agreement with the results of [3], obtained within the Fermi Hyper Netted Chain approach. Using the same cluster expansion with wave function parameters wich minimize the ground state energy the semi-inclusive reaction of type $A(e,e'p)X$ has been calculated taking final state interaction effects into account within a Glauber type calculation [5]; the comparison between the resulting distorted and undistorted momentum distributions provides an estimate of the transparency of the nuclear medium to the propagation of the hit proton. The effect of color transparency has also been considered in particular by the Finite Formation Time approach of [6]; it is shown that at high values of Q^2 the effects of FFT strongly reduce the final state interaction consistently with the idea of a reduced interaction of the hadron produced inside the nucleus [7].

1 - CALCULATING GROUND STATE PROPERTIES OF COMPLEX NUCLEI

- The nuclear wave function is

$$|\Psi\rangle \;=\; \hat{F}\,|\phi\rangle \;=\; \prod_{i<j}^{A} \hat{f}_{ij}\,|\phi\rangle$$

where $\hat{f}_{ij}$ is the state-dependent correlation operator

$$\hat{f}_{ij} \;=\; \sum_n f^{(n)}(r_{ij})\,\hat{O}^{(n)}_{ij}$$

and ϕ is a shell model, Slater determinant wave function.

- A proper cluster expansion for a given operator $\hat{O}$

$$\langle\hat{O}\rangle \;=\; \frac{\langle\Psi|\hat{O}|\Psi\rangle}{\langle\Psi|\Psi\rangle} \;=\; \frac{\langle\phi|\hat{F}^\dagger\hat{O}\hat{F}|\phi\rangle}{\langle\phi|\hat{F}^\dagger\hat{F}|\phi\rangle}\;;$$

has been developed;

- expanding

$$\hat{F} = \prod_{i<j}\hat{f}^{\,2}(ij) = \prod_{i<j}\Big(1+\hat{\eta}(ij)\Big) = 1+\sum_{i<j}\hat{\eta}(ij)+\sum_{ij<lk}\hat{\eta}(ij)\hat{\eta}(lk)+\ldots$$

where $\hat{\eta}(ij) \;=\; \hat{f}^2(ij) - 1$;one obtains, up to second order in $e\hat{t}a$, the expression for $\langle\hat{O}\rangle$:

$$\begin{aligned}
\mathcal{O}_0 \;&\equiv\; \langle\hat{\mathcal{O}}\rangle\;,\\
\mathcal{O}_1 \;&=\; \langle\sum_{ij}\hat{\eta}_{ij}\,\hat{\mathcal{O}}\rangle \;-\; \mathcal{O}_0\,\langle\sum_{ij}\hat{\eta}_{ij}\rangle\;,\\
\mathcal{O}_2 \;&=\; \langle\sum_{ij<kl}\hat{\eta}_{ij}\,\hat{\eta}_{kl}\,\hat{\mathcal{O}}\rangle \;-\; \langle\sum_{ij}\hat{\eta}_{ij}\,\hat{\mathcal{O}}\rangle\,\langle\sum_{ij}\hat{\eta}_{ij}\rangle\\
&\quad -\,\mathcal{O}_0\left(\langle\sum_{ij<kl}\hat{\eta}_{ij}\,\hat{\eta}_{kl}\,\rangle \;-\; \langle\sum_{ij}\hat{\eta}_{ij}\rangle^2\right)\;;
\end{aligned}$$

where

$$\langle\,\ldots\,\rangle \;\equiv\; \langle\phi|\;\ldots\;|\phi\rangle\;.$$

is the expectation value over the shell model wave function.

1.1 - THE GROUND STATE ENERGY

- The expectation value of the many-body hamiltonian

$$\hat{H} \;=\; \hat{T} + \hat{V} \;=\; -\frac{\hbar^2}{2\,m}\sum_{i=1}^{A}\hat{\nabla}_i^2 \;+\; \frac{1}{2}\sum_{i<j}\hat{v}_{ij}$$

is obtained in terms of the *one-body mixed* and *two-body* density matrices, according to:

$$\langle\hat{T}\rangle \;=\; \int d\boldsymbol{k}\,k^2\,n(\boldsymbol{k})\;,$$

with

$$n(\boldsymbol{k}) \;=\; \frac{1}{(2\pi)^3}\int d\boldsymbol{r}d\boldsymbol{r}'\,e^{i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\,\rho^{(1)}(\boldsymbol{r},\boldsymbol{r}')\,,$$

and

$$\langle\hat{V}\rangle \;=\; \sum_n\int d\boldsymbol{r}_1\,d\boldsymbol{r}_2\,v^{(n)}(r_{12})\,\rho^{(2)}_{(n)}(\boldsymbol{r}_1,\boldsymbol{r}_2)\;\;;$$

where the index n denotes the corresponding operator of the realistic potential:

$$\hat{V} \;=\; \frac{1}{2}\sum_{i<j}\hat{v}(ij) \;=\; \frac{1}{2}\sum_{i<j}\sum_n v^{(n)}(r_{ij})\,\hat{O}^{(n)}_{ij}$$

1.2 - DENSITIES & MOMENTUM DISTRIBUTIONS

- Using the *Argonne V8′* two-body interaction and the correlation functions from

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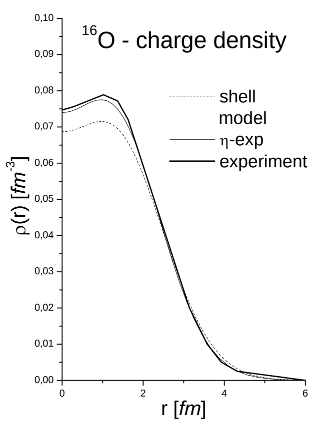
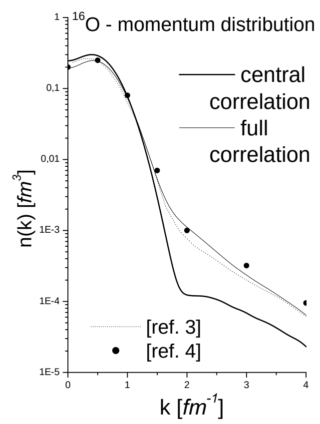
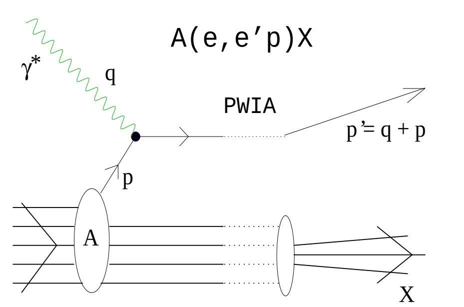
we have calculated the binding energy, momentum distributions, charge densities and mean radii of light and medium-weight nuclei, finding good agreement with experimental data.

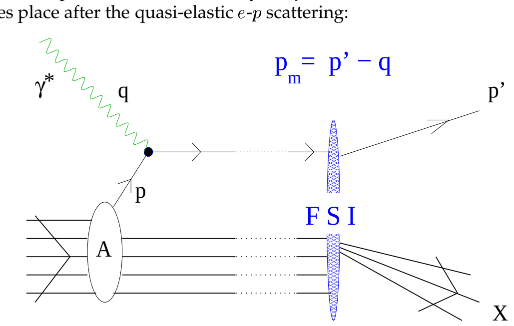




2 - FINAL STATE INTERACTION IN THE A(e,e′p)X PROCESS



- the PWIA picture is invalidated by the *final state interaction* (FSI), which takes place after the quasi-elastic e-p scattering:



where $\boldsymbol{p}_m = \boldsymbol{p}' - \boldsymbol{q}$ is the *missing momentum* of the process.

2.1 - THE DISTORTED MOMENTUM DISTRIBUTION WITHIN THE GLAUBER APPROACH

- within the factorization approximation, the cross section for the process $A(e,e'p)X$ reads as follows:

$$\frac{d\sigma}{dQ^2\,d\nu\,d\boldsymbol{p}'} \;=\; K\,\sigma_{ep}\,P_D(E_m,\boldsymbol{p}_m)$$

with

$$\boldsymbol{p}_m = \boldsymbol{p}' - \boldsymbol{q}\;;$$

- the distorted nucleon *spectral function* can be expressed as

$$P_D \propto \sum_f\Big|\langle\boldsymbol{p}_m\,S\,\psi_f^{A-1}\,|\psi_o^A\rangle\Big|^2\;\delta(E_m - E)$$

where $E \;=\; E_{A-1} - E_A$ and S takes FSI into account; summing over the final states:

$$\sum_f|\psi_f^{A-1}\rangle\langle\psi_f^{A-1}| \;=\; 1\;\;,$$

one obtains the distorted momentum distribution:

$$n_D(\boldsymbol{p}_m) \;=\; \int d\boldsymbol{r}d\boldsymbol{r}'\,e^{i\,\boldsymbol{p}_m\cdot(\boldsymbol{r}-\boldsymbol{r}')}\,\rho_D(\boldsymbol{r},\boldsymbol{r}')$$

- Within the Glauber approach the factor S reads:

$$S \;=\; \prod_{i<j}G(ij) \;=\; \prod_{i<j}\,(1\;-\;\theta(z_j - z_i)\Gamma(\boldsymbol{b}_i - \boldsymbol{b}_j))\;;$$

- the profile function

$$\Gamma(\boldsymbol{b}) = \frac{\sigma_{tot}}{4\pi B}(1\;-\;i\,\alpha)e^{-\boldsymbol{b}^2/2B^2}$$

describes the p-N scattering.

- The distorted one-body mixed density matrix is obtained by cluster expanding

$$\rho_D \;=\; \frac{\langle\phi|\hat{F}^\dagger\,S^\dagger\;\hat{\rho}\;S\,\hat{F}|\phi\rangle}{\langle\phi|\hat{F}^\dagger\,\hat{F}|\phi\rangle}\;.$$

The distorted momentum distribution is then obtained:

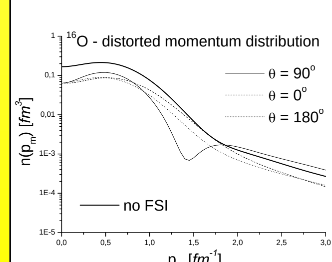
$$n_D(\boldsymbol{p}_m) \;=\; \int d\boldsymbol{r}d\boldsymbol{r}'\,e^{i\,\boldsymbol{p}_m\cdot(\boldsymbol{r}-\boldsymbol{r}')}\,\rho_D(\boldsymbol{r},\boldsymbol{r}')\;\;;$$

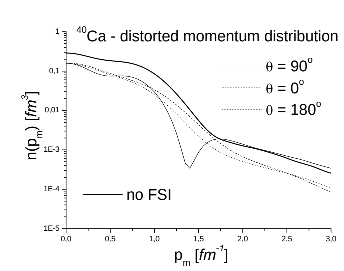
- the nuclear transparency is defined as:

$$T \;=\; \int d\boldsymbol{p}_m\,n_D(\boldsymbol{p}_m)/\int d\boldsymbol{p}_m\,n(\boldsymbol{p}_m)$$

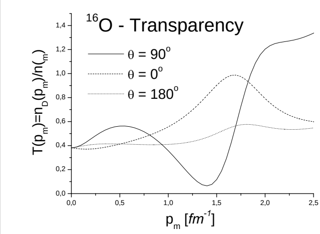
2.2 - THE DISTORTED MOMENTUM DISTRIBUTION

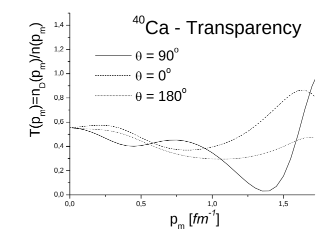
- As a result of the introduction of $\hat{G}(\boldsymbol{b}_{ij}, z_{ji})$ the distorted momentum distribution will depend upon the *vector* $\boldsymbol{p}_m$ and therefore will look different for different values of the angle θ between $\boldsymbol{p}_m$ and $\boldsymbol{q}$





2.3 - THE NON-INTEGRATED NUCLEAR TRANSPARENCY





3 - CALCULATION OF FINITE FORMATION TIME EFFECTS

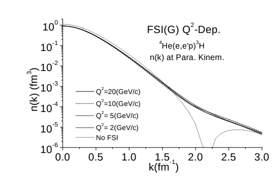
- The FFT in the nuclear medium is included by replacing the Glauber operator [7] by

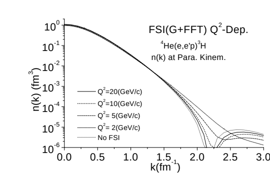
$$G(ij) \;=\; 1\;-\;J(z_j - z_i)\,\Gamma(\boldsymbol{b}_i - \boldsymbol{b}_j)$$

where

$$J(z) \;=\; \theta(z)\,(1\;-\;e^{-\,z\,\frac{x\,m\,M^2}{Q^2}})\;;$$

with x the Björken variable and $M^2 = (m^*)^2 - m^2$; m^* is an average excitation mass.





- with the inclusion of FFT effect the un-distorted momentum distribution is recovered, provided Q^2 is sufficiently high.

CONCLUSIONS

- We have developed a linked cluster expansion to calculate the ground state properties of complex nuclei
- the binding energy, densities and momentum distributions have been obtained for ^{16}O and ^{40}Ca
- the cluster expansion has been generalized to take account of the final state interactions in A(e,e′p)X by the Glauber multiple scattering approach
- we can calculate by this procedure the distorted momentum distributions and nuclear transparency for complex nuclei

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