

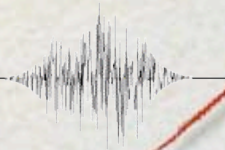
# Nonlinear Energy Harvesting

**Helios Vocca**

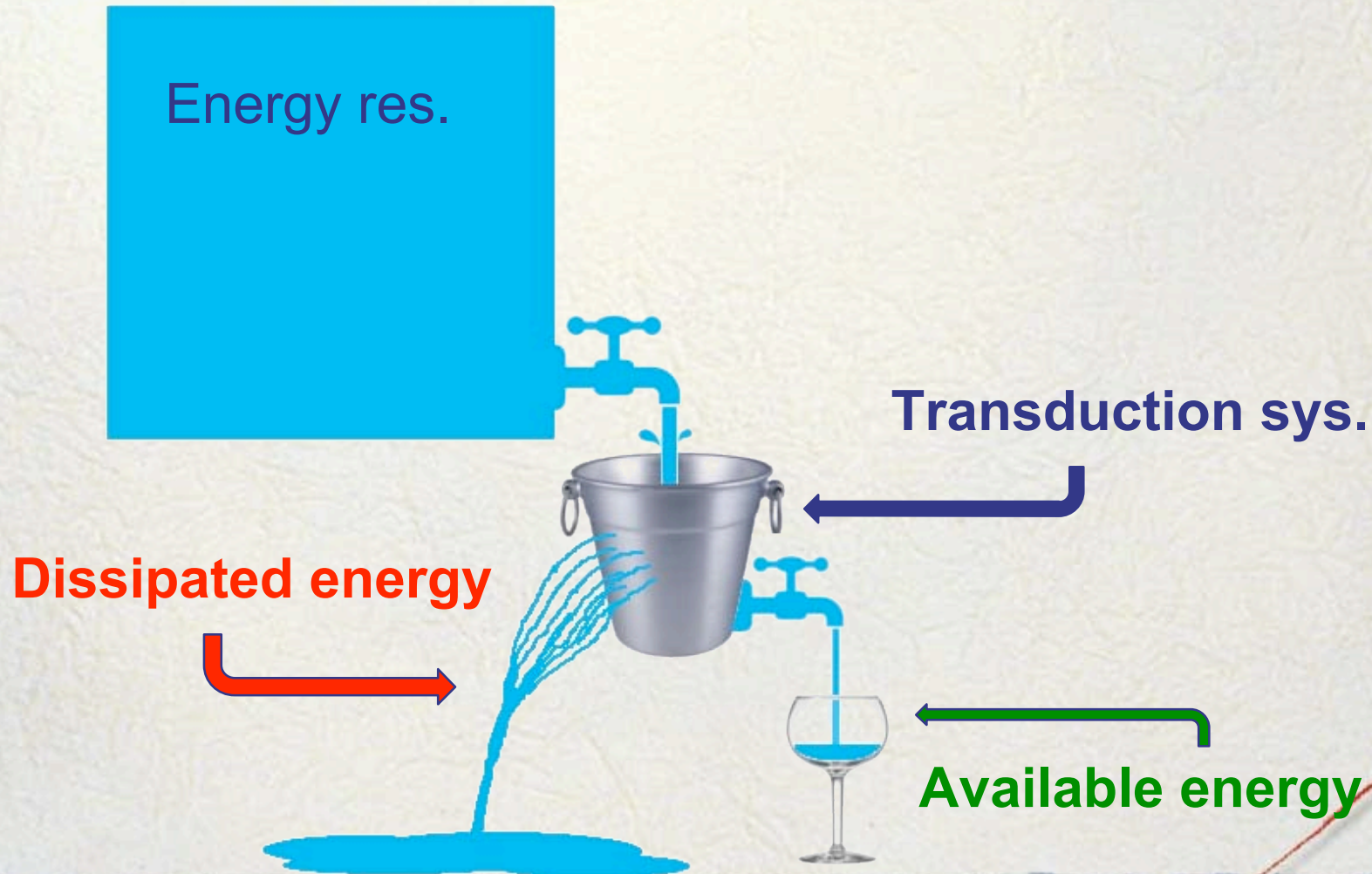
NiPS Laboratory, Dipartimento di Fisica  
Università degli Studi di Perugia, Italy  
and Wisepower srl

[www.nipslab.org](http://www.nipslab.org)

**N.i.P.S** Laboratory  
Noise in Physical Systems



# Energy harvesting basic ideas



# Energy harvesting basic ideas

## Kinetic energy

wind

sound

Falling bodies

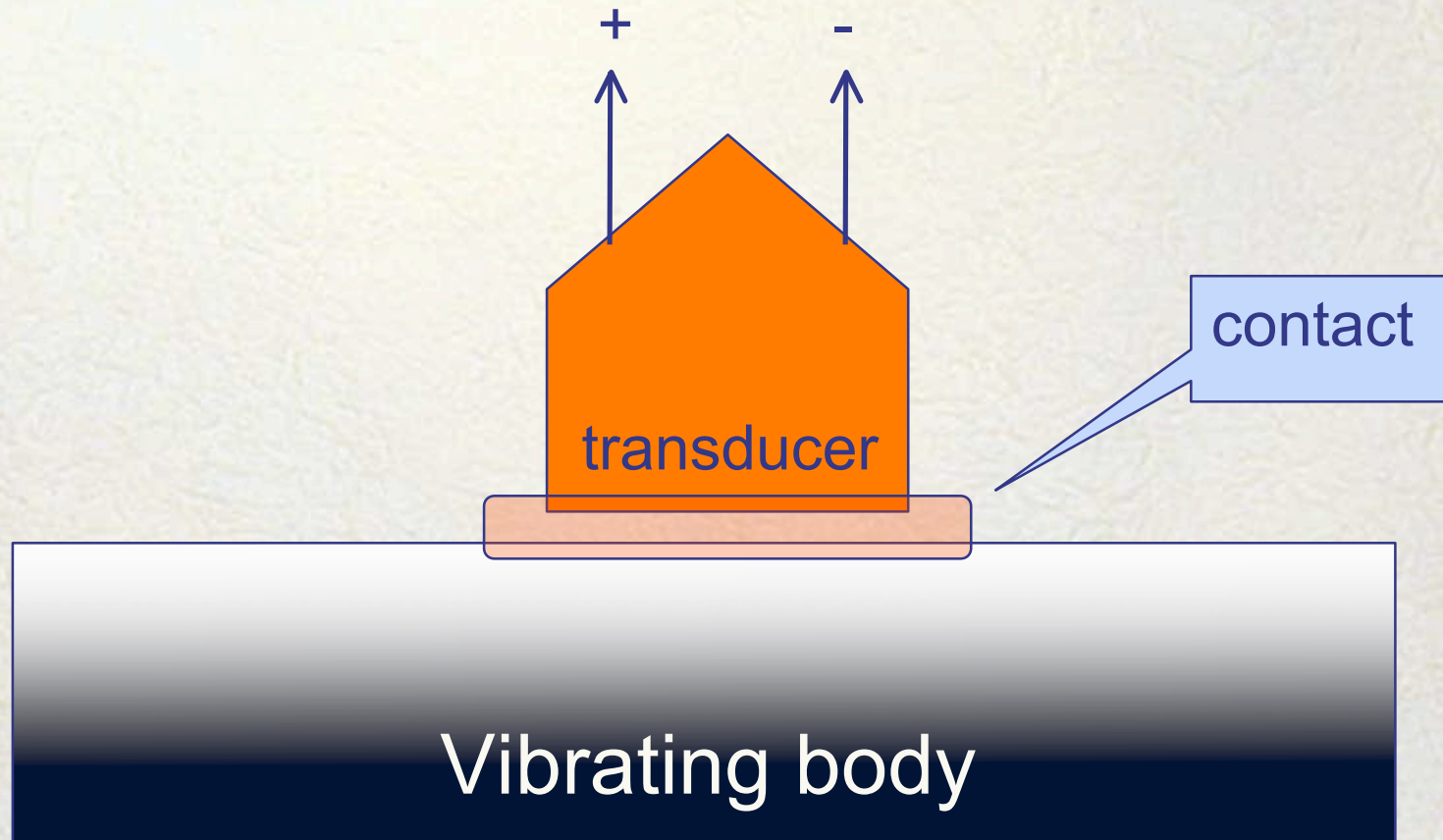
vibrations

water waves and tides

Focus on **random vibrations** of solid bodies....

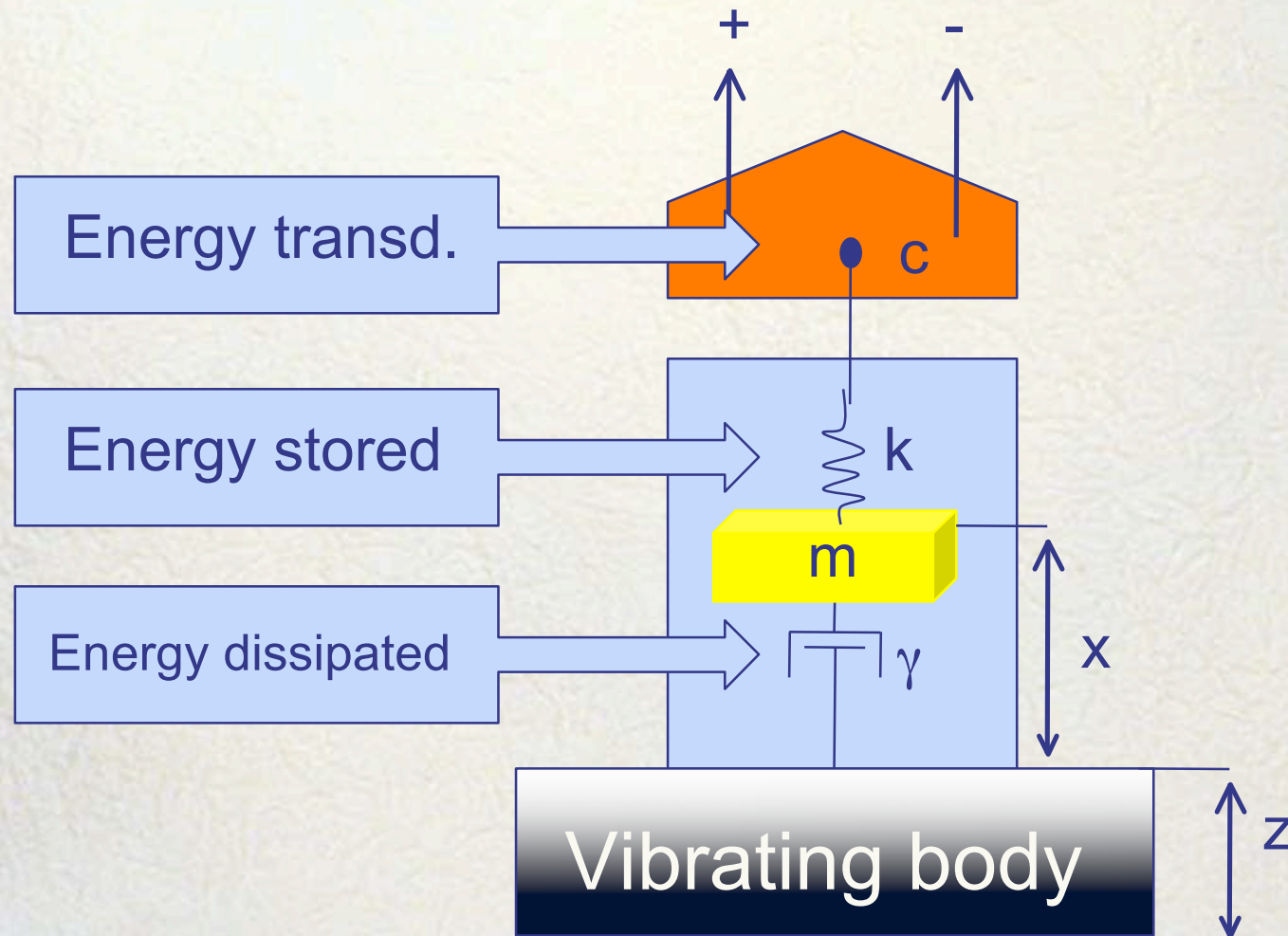
# Noise energy harvesting

## Basic Scheme

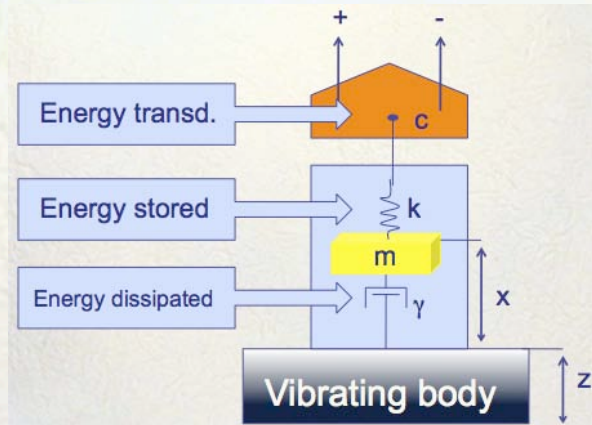


# Noise energy harvesting

## Dynamical model



# Noise energy harvesting



Dynamical model

Details depend on the physics...

$$\left. \begin{aligned}
 m\ddot{x} &= \frac{dU(x)}{dx} \\
 \dot{V} &= F(\dot{x}, V)
 \end{aligned} \right\} \# \dot{x} \quad c(x, V) + z$$

Equations that link the vibration-induced displacement with the Voltage

# Noise energy harvesting

## Transduction mechanisms

1

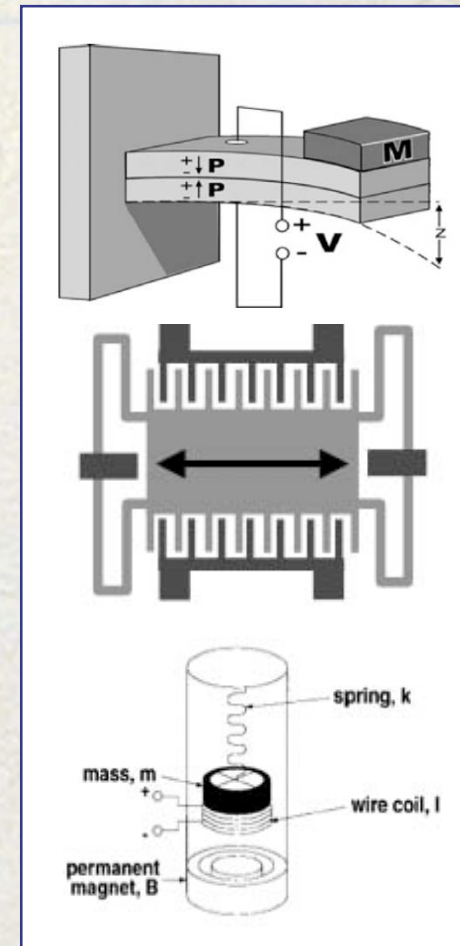
Piezoelectric: dynamical strain is converted into voltage difference.

2

Capacitive: geometrical variations induce voltage difference

3

Inductive: dynamical oscillations of magnets induce electric current in coils



# Noise energy harvesting

## Transduction mechanisms

1

Piezoelectric: dynamical strain is converted into voltage difference.

$$m\ddot{x} = -\frac{dU(x)}{dx} - \gamma\dot{x} - K_V V + \xi_z$$

$$\dot{V} = K_c \dot{x} - \frac{1}{\tau_p} V$$

The Physics of piezo materials

The available power is proportional to  $V^2$

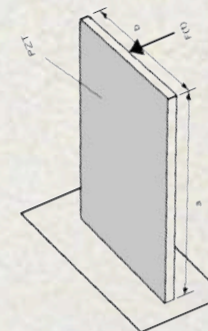
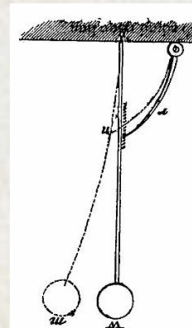
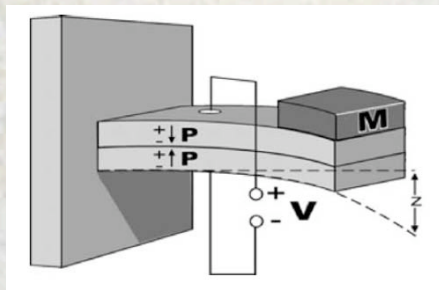
# Noise energy harvesting

## Linear systems

When  $U(x) = \frac{1}{2} kx^2$  it is called a linear system

Linear systems have some interesting features... (and engineers like them most)

- 1) There exist a simple math theory to solve the eq.s
- 2) They have a resonant behaviour (resonance freq.)
- 3) They can be “easily” realized with catilevers and pendula



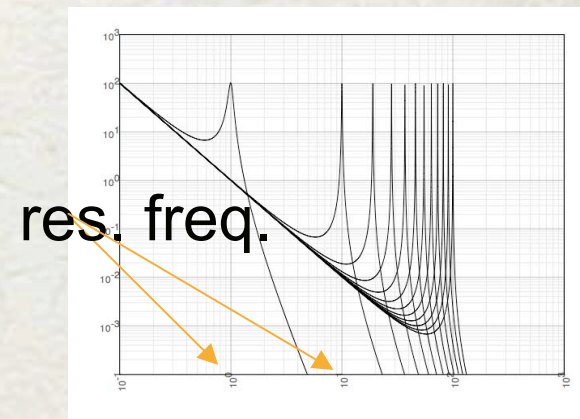
# Noise energy harvesting

## Linear systems

For a linear system the transfer function presents one or more peaks corresponding to the resonance frequencies and thus it is efficient mainly when the incoming energy is abundant in that regions...

This is a serious limitation when you want to build a small energy harvesting system...

**Why ?!!**



# Noise energy harvesting

## Linear systems

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For two main reasons...

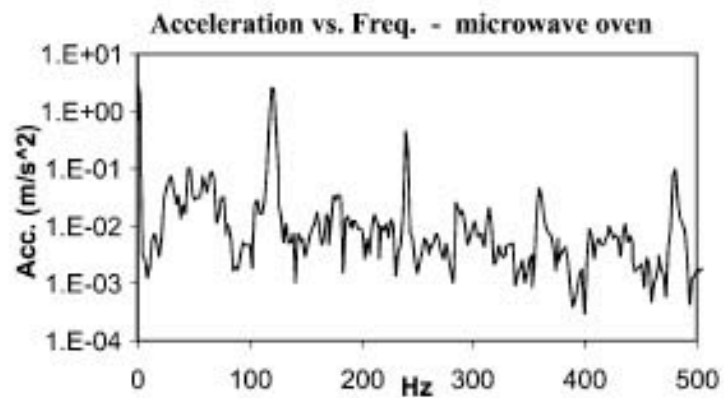
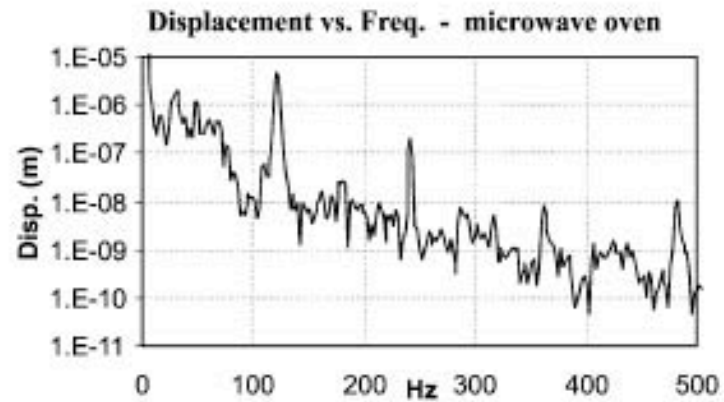
- (1) the frequency spectrum of available vibrations instead of being sharply peaked at some frequency is usually very broad.
- (2) The frequency spectrum of available vibrations is particularly rich in energy in the low frequency part... and it is very difficult, if not impossible, to build small low-frequency resonant systems...

Let's see some examples...

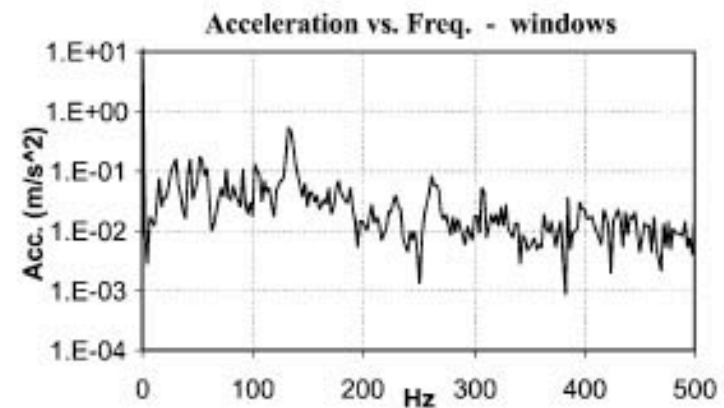
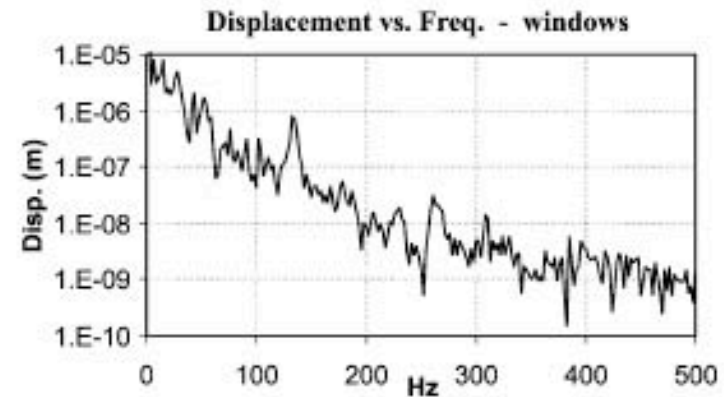
# Noise energy harvesting

*S. Roundy et al. / Computer Communications 26 (2003) 1131–1144*

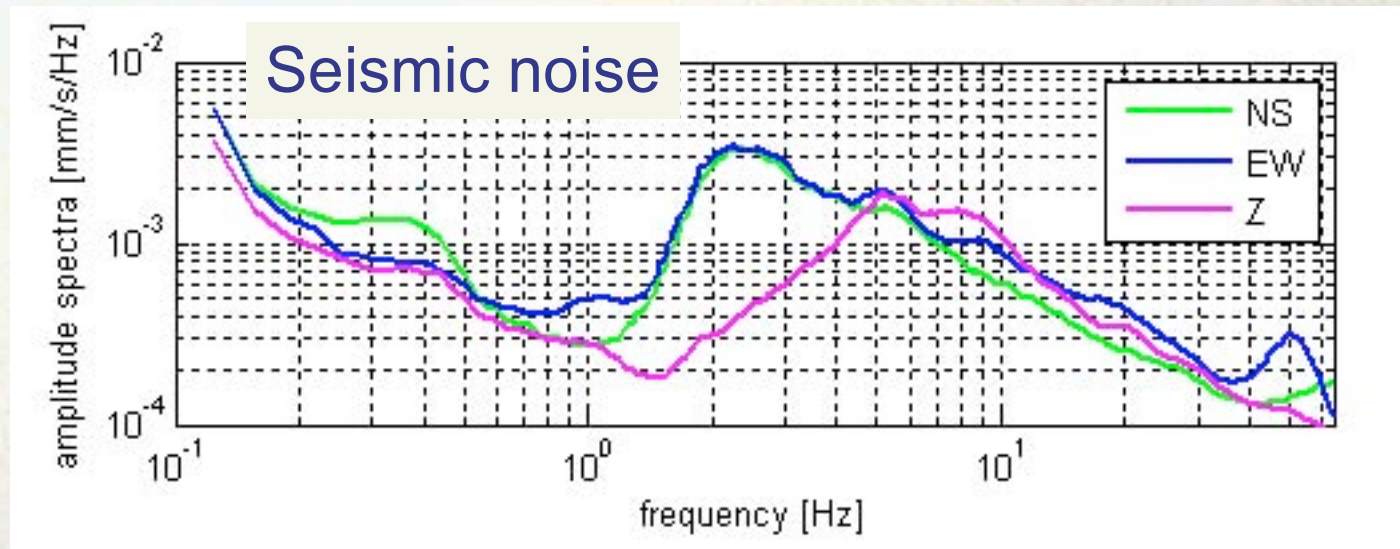
**Microwave Casing**



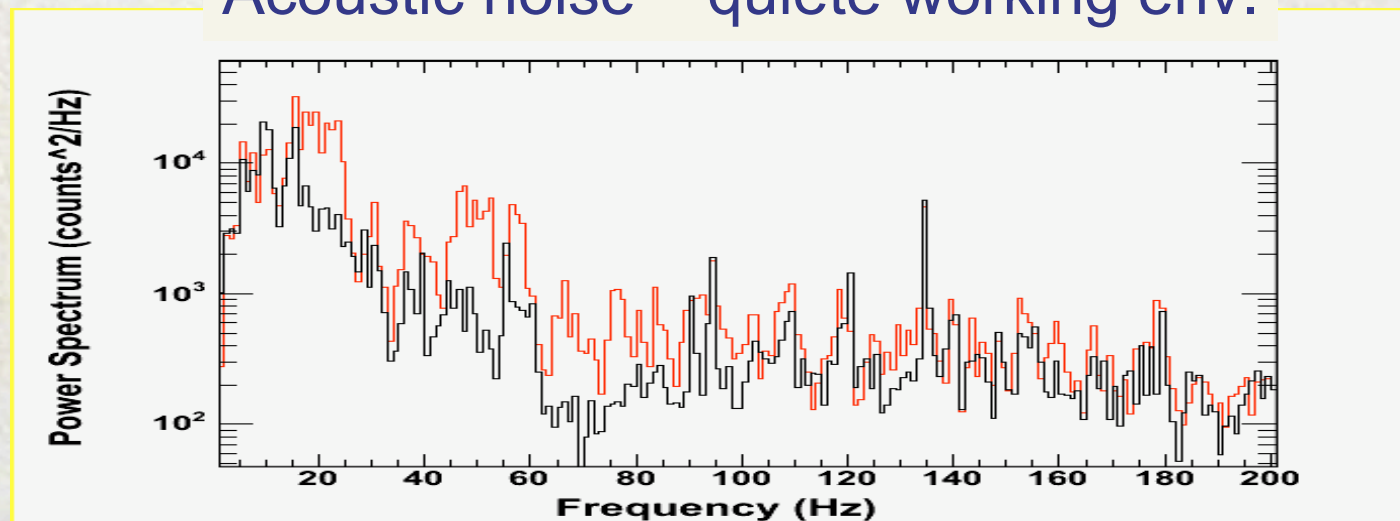
**Windows Next to a Busy Street**



# Noise energy harvesting

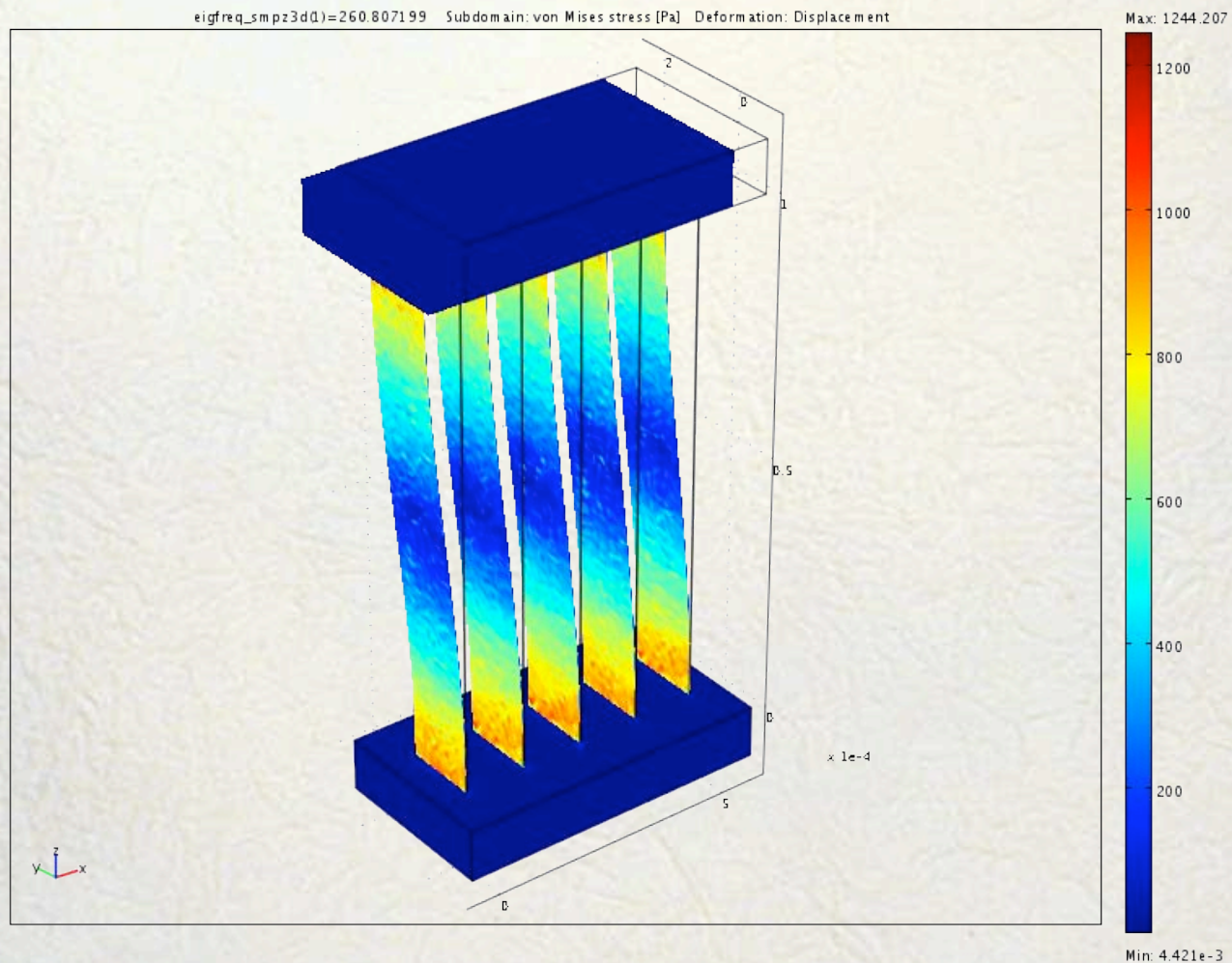


## Acoustic noise – quiete working env.



# Noise energy harvesting

Micro energy harvesting system...



# Noise energy harvesting

Wish list for the perfect vibration harvester

- 1) Capable of harvesting energy on a broad-band
- 2) No need for frequency tuning
- 3) Capable of harvesting energy at low frequency



- 1) Non-resonant system
- 2) "Transfer function" with wide frequency resp.
- 3) Low frequency operated

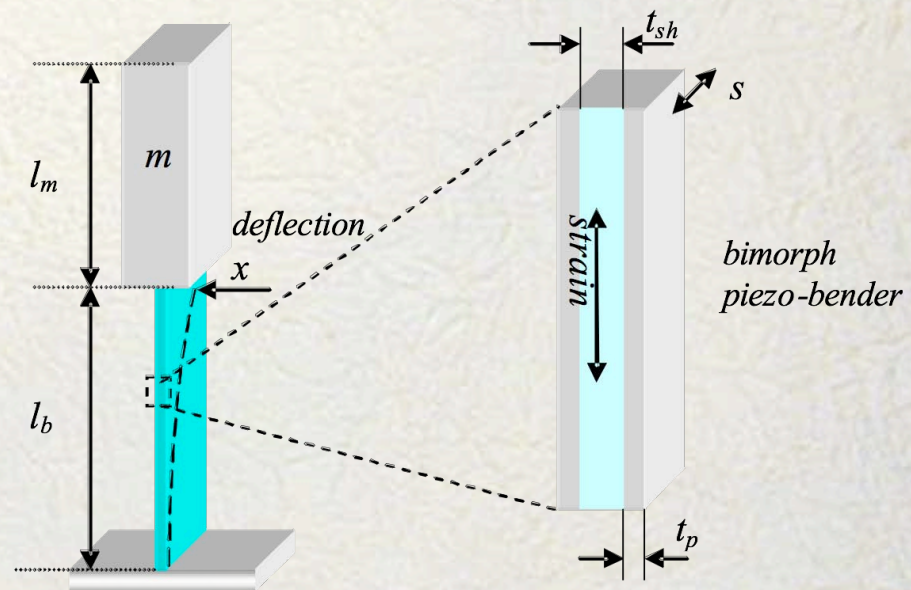
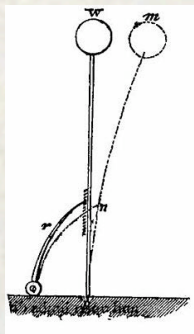
# Noise energy harvesting

## NON-Linear mechanical oscillators

- 1) Non-resonant system
- 2) "Transfer function" with wide frequency resp.
- 3) Low frequency operated

Example...

Inverted pendulum

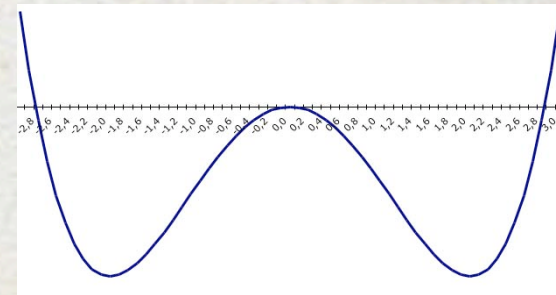
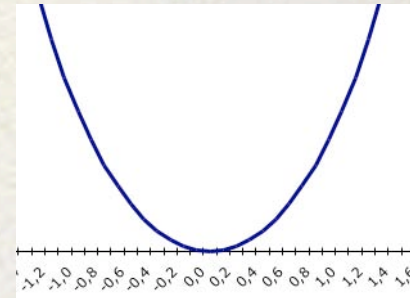
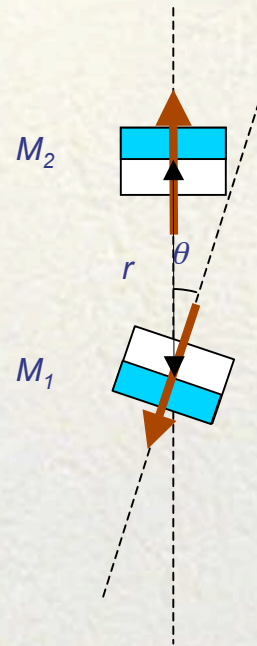
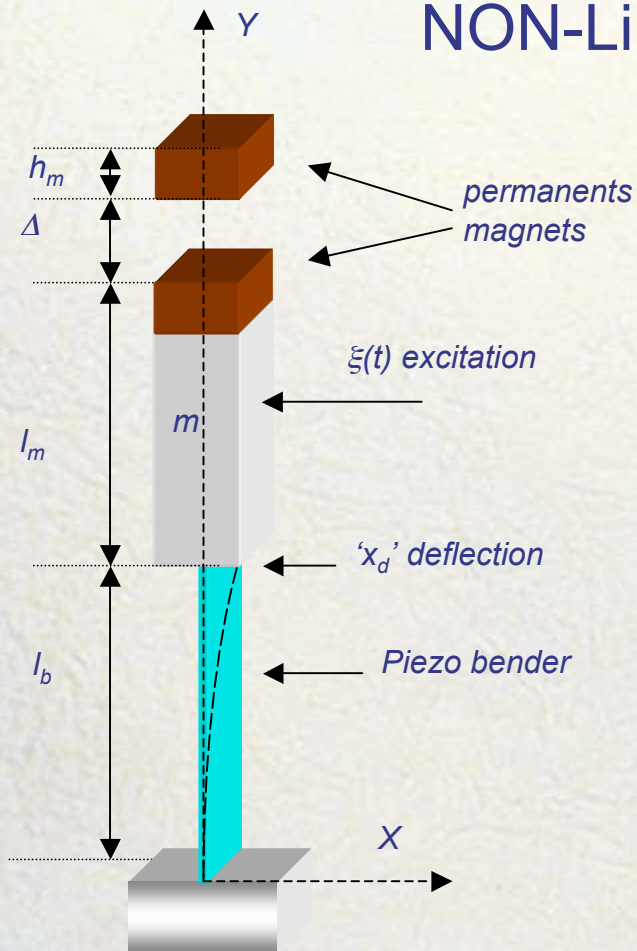


F. Cottone, PhD Thesis, Perugia 2007

# Noise energy harvesting

## NON-Linear mechanical oscillators

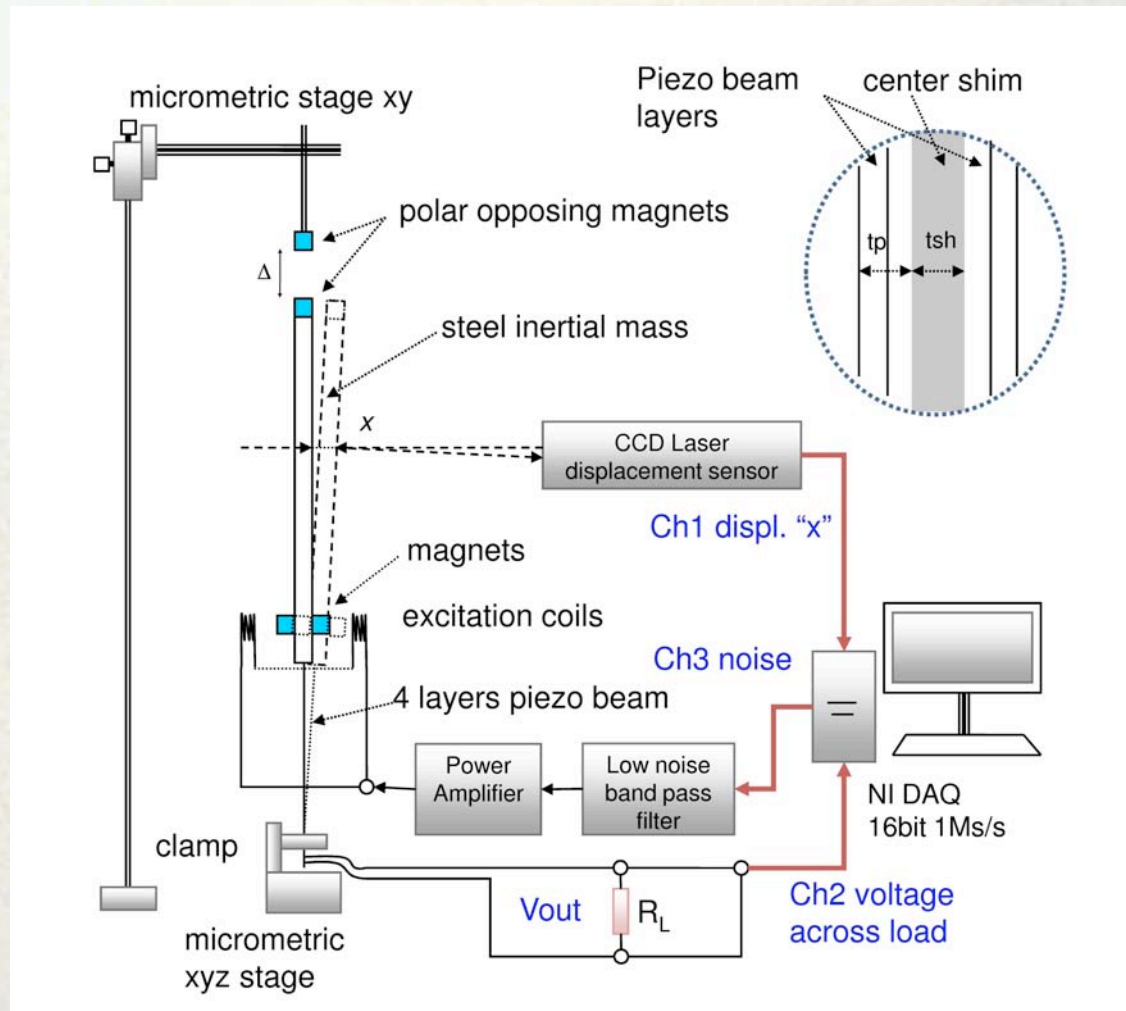
### NON-Linear Inverted pendulum



b)

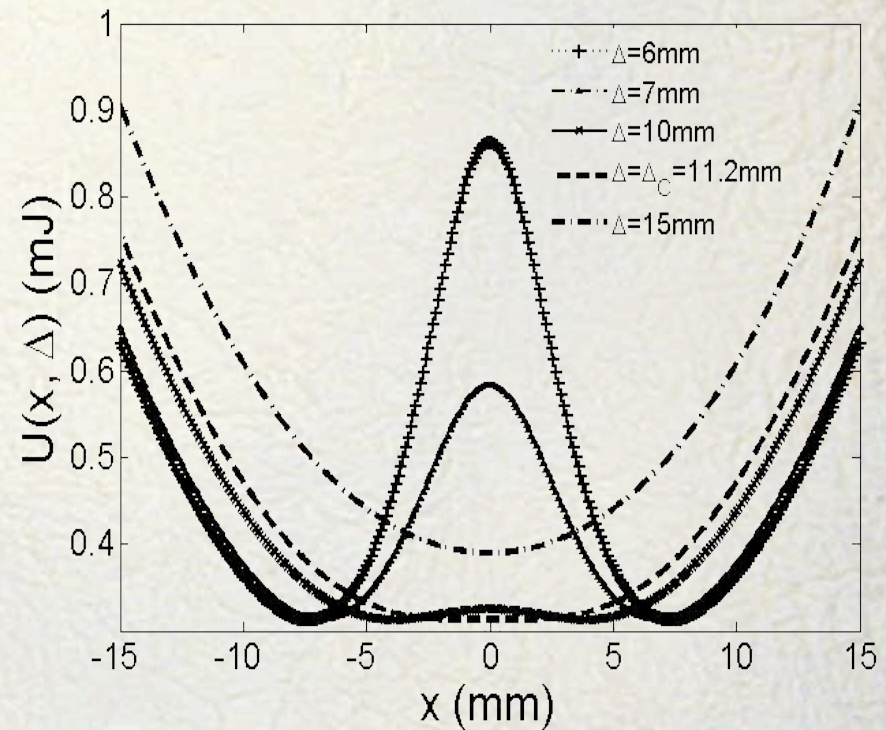
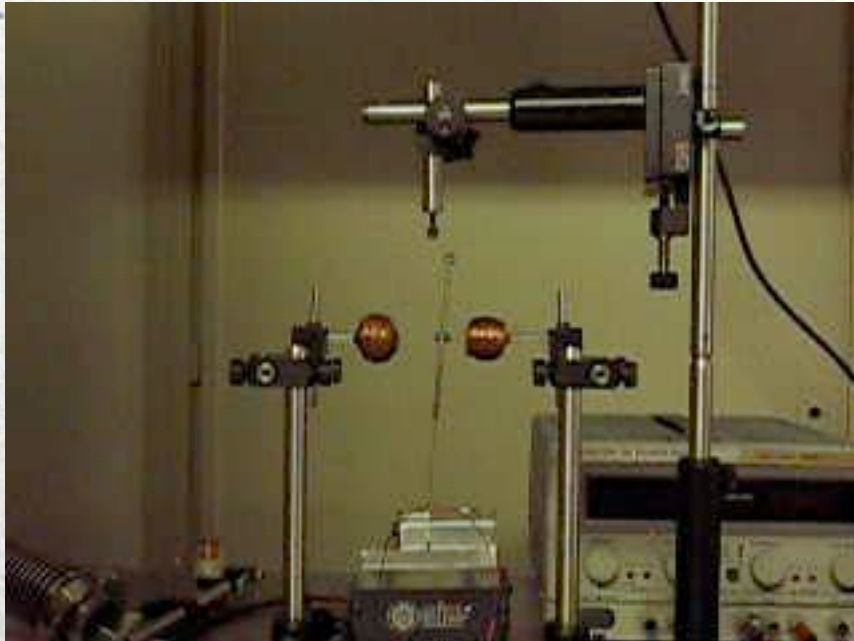
# Noise energy harvesting

## NON-Linear mechanical oscillators



# Noise energy harvesting

## NON-Linear mechanical oscillators

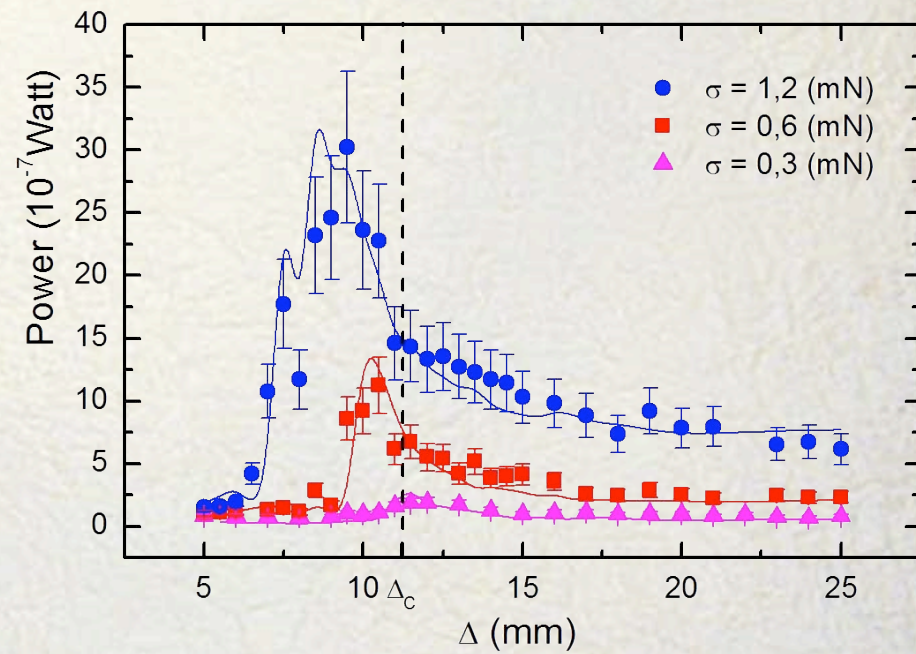
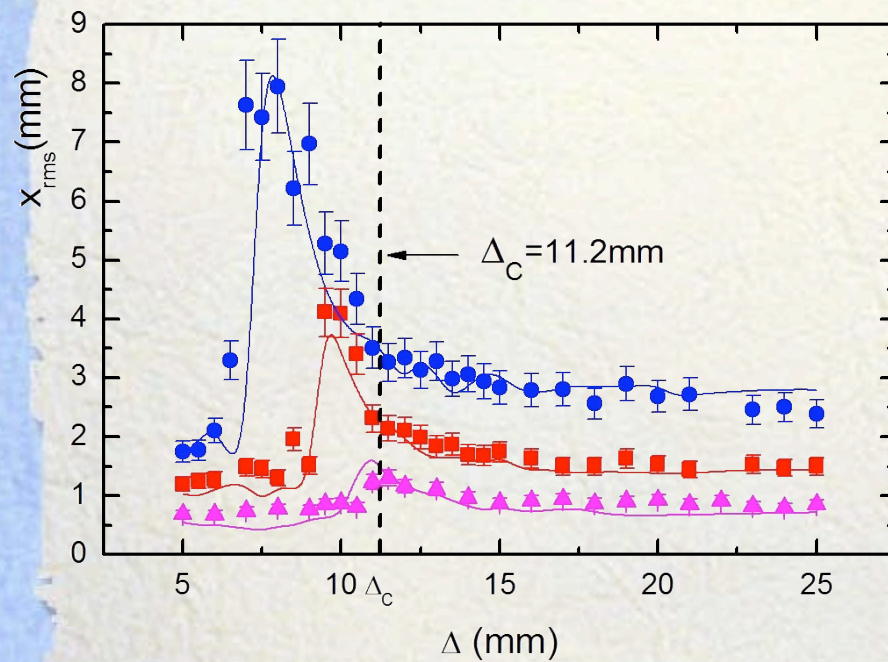


<http://www.nipslab.org/node/1676>

Nonlinear Energy Harvesting, F. Cottone; H. Vocca; L. Gammaitoni  
**Physical Review Letters**, 102, 080601 (2009)

# Noise energy harvesting

## NON-Linear mechanical oscillators

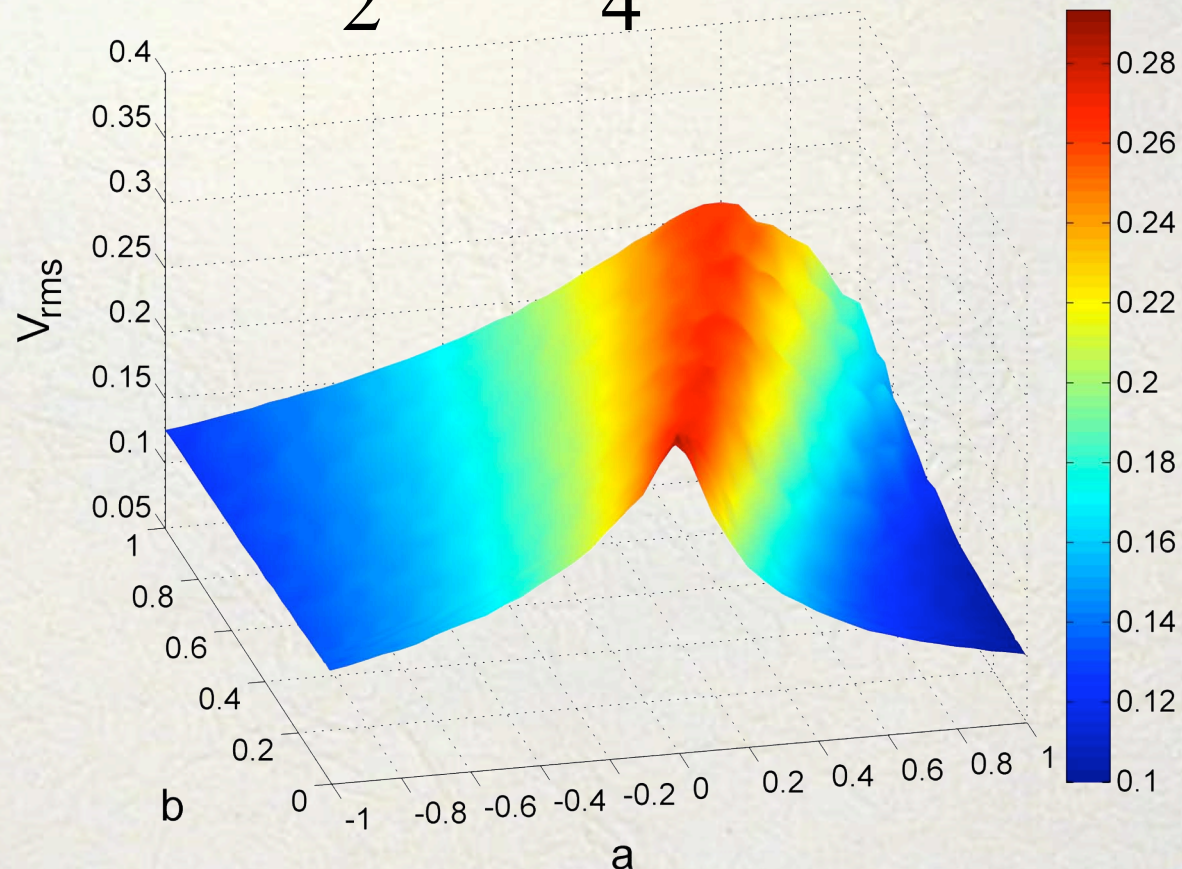


# Noise energy harvesting

## Non-linear systems

$$U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$$

Duffing potential



# Noise energy harvesting

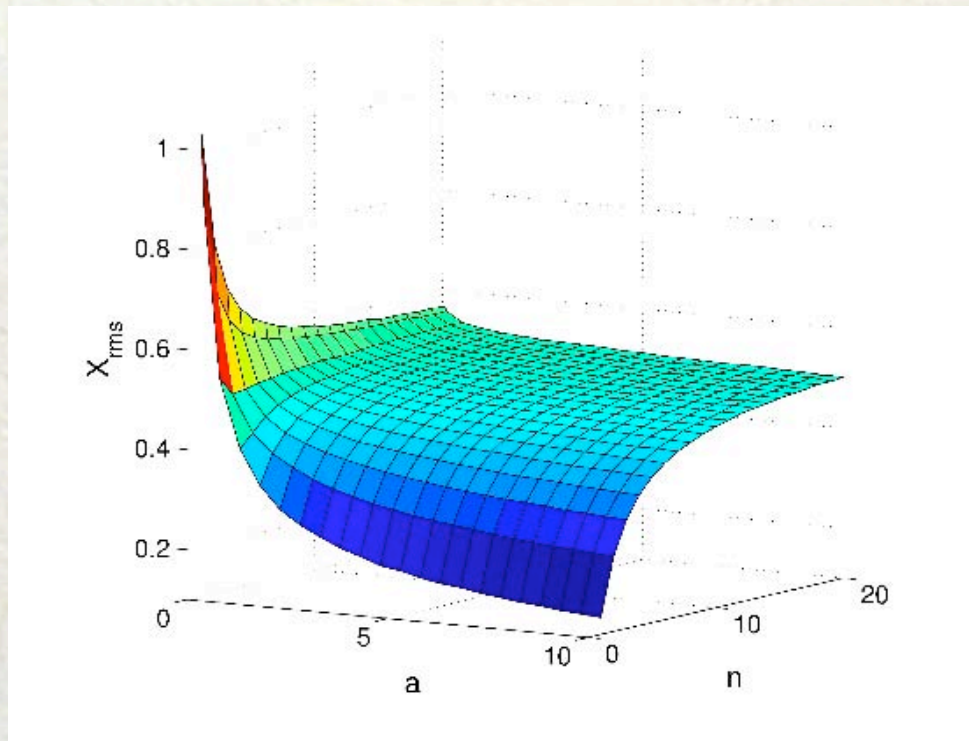
Only bistability???

A more general monostable potential...

$$U(x) = ax^{2n}$$

with :  $a > 0$

$n = 1, 2, \dots$



$$a_{th} \approx \frac{D}{4} = \sigma^2 \tau$$

## To think about...

- 1) Non resonant (i.e. non-linear) mechanical oscillators can outperform resonant (i.e. linear) ones\*
- 2) Non-linear systems are more difficult to treat
- 3) Bistability is not the only nonlinearity available...

\* **wisepower technology** patent. For more info see: [www.nipslab.org](http://www.nipslab.org), [www.wisepower.it](http://www.wisepower.it)