Elastic proton electron scattering

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« Scattering and annihilation electromagnetic processes »

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Motivations

Elastic ep scattering is a privileged tool for learning on the internal structure of the proton.
Unpolarized cross section, M. N. Rosenbluth (1950)
Polarization method by A.I. Akhiezer and M.P. Rekalo (1967).

Possible applications of pe scattering (inverse kinematics):

- Proton charge radius measurement
- Polarized (anti)proton beams ($\bar{p}\, e^{+}$)
- Beam polarimeters for high energy polarized proton beams, Novisibirsk (1997).

Plan

- Formalism
- Kinematics and proton charge radius
- Polarization observables:
  - Transfer coefficients $p + \vec{e} \longrightarrow \vec{p} + e$
  - Correlation coefficient $\vec{p} + \vec{e} \longrightarrow p + e$
- Summary

Inverse Kinematics (Lab.)

$$p(p_1) + e(k_1) \rightarrow p(p_2) + e(k_2)$$

- Inverse kinematics : projectile heavier than the target →take into account the electron mass

- Specific kinematics:
  - very small scattering angles
  - very small transferred momenta

- Equivalent total energy, $\boldsymbol{s} = (\boldsymbol{p}_1 + \boldsymbol{k}_1)^2$:

Proton beam energy

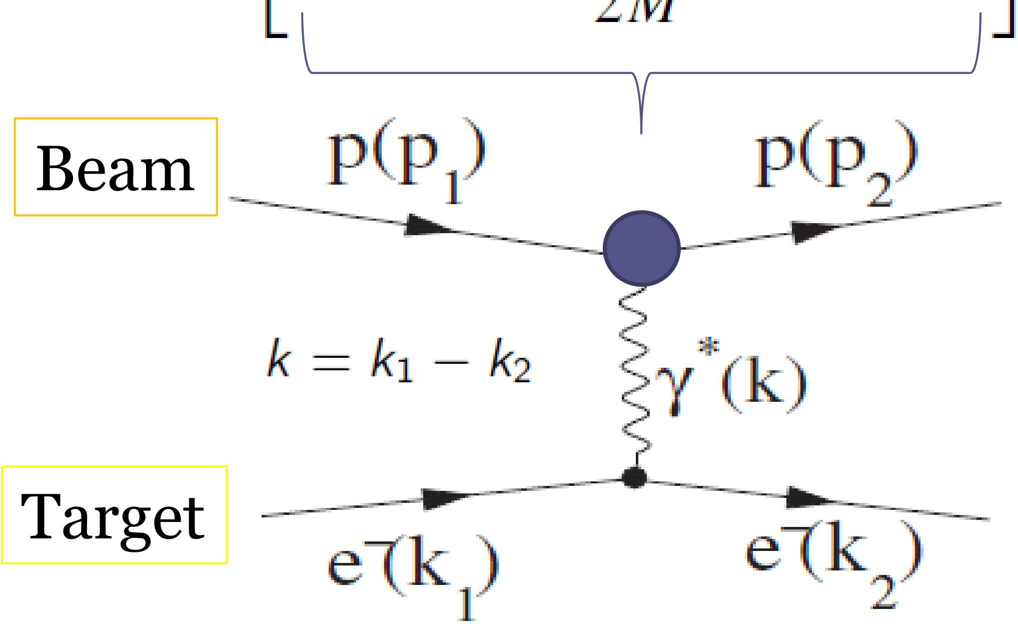
$$E_p = \frac{M_p}{m_e}\ \varepsilon_e \sim 2000\ \varepsilon_e$$

electron beam energy

Formalism

- Scattering amplitude : $\mathcal{M} = \frac{e^2}{k^2} j_\mu J_\mu$
- Hadronic current:

$$J_\mu = \bar{u}(p_2)\left[F_1(k^2)\gamma_\mu - \frac{1}{2M}F_2(k^2)\sigma_{\mu\nu}k_\nu\right]u(p_1)$$



Sachs form factors:

$$G_M(k^2) = F_1(k^2) + F_2(k^2)$$

$$G_E(k^2) = F_1(k^2) - \tau F_2(k^2)$$

$$\tau = -k^2/(4M^2)$$

- Leptonic current: $j_\mu = \bar{u}(k_2)\gamma_\mu u(k_1)$

Unpolarized cross section (lab.)

$$\frac{d\sigma}{dk^2} = \frac{1}{64\pi^2}\frac{\overline{|\mathcal{M}|^2}}{m^2\vec{p}^2} = \frac{\pi\alpha^2}{2m^2\vec{p}^2}\frac{\mathcal{D}}{k^4},$$

$$\mathcal{D} = k^2(k^2+2m^2)G_M^2(k^2)+2\left[k^2M^2+\frac{1}{1+\tau}\left(2mE+\frac{k^2}{2}\right)^2\right]\left[G_E^2(k^2)+\tau G_M^2(k^2)\right]$$

$\tau = -k^2/(4M^2)$

M (m): Proton (electron) mass,
E: energy of incident proton beam.

- Diverges as k^{-4}
- Dominance of G_E at low $Q^2 = -k^2$.

Precise measurement of proton charge radius

Proton charge radius

- For small values of $Q^2 = -k^2$:

$$G_E(Q^2) = 1 - \frac{1}{6}Q^2 < r_c^2 > +O(Q^2)$$

$$< r_c^2 >= -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

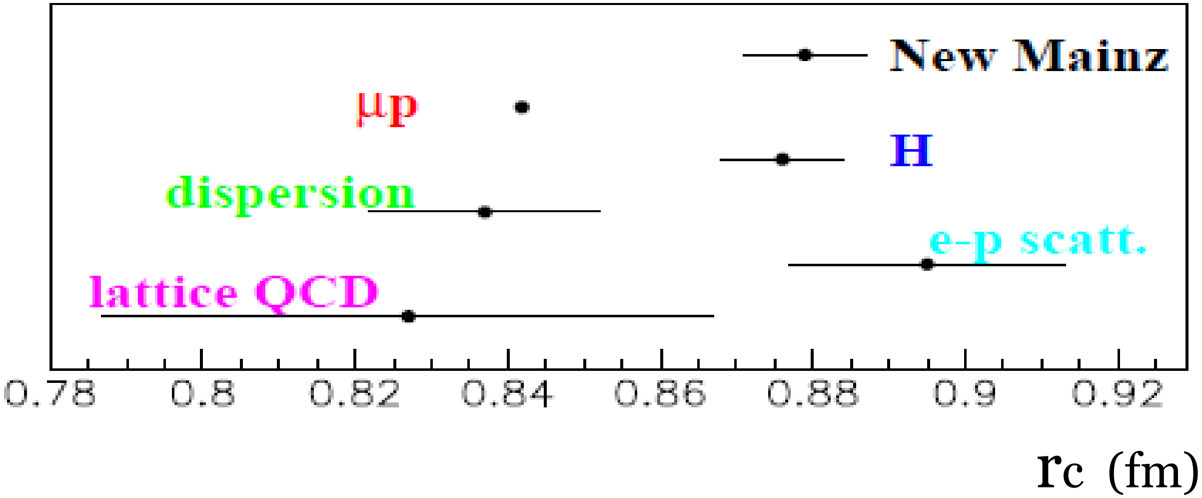
- (Muonic) Hydrogen spectroscopy (Lamb shift):

$$\Delta E^{FS} = \frac{2(Z\alpha)^4}{3n^3} m_r^3 \underline{r_c^2} \delta_{l0}$$

Proton structure correction to the energy levels of atomic electron

Proton radius puzzle:

J. Phys. Conf. Ser. **312**, 032002 (2011)



$r_c = 0.879(8)$

$r_c = 0.84184(67)$

$r_c = 0.8768(69)$

$r_c = 0.895(18)$

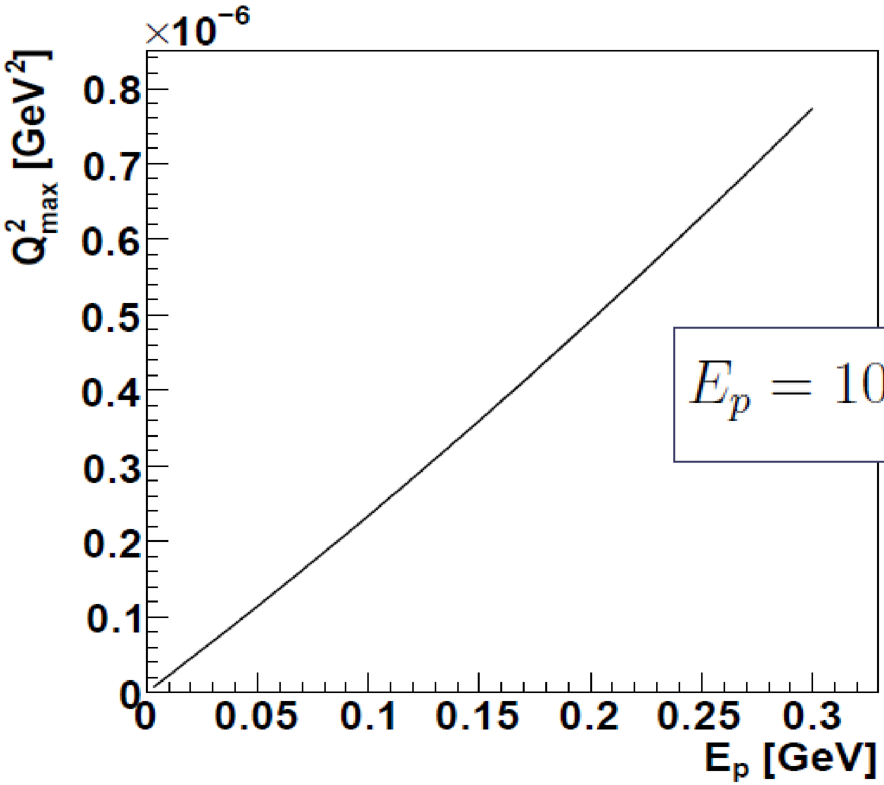
In ep scattering (● ●), precision on the measurement is strongly related to the fit function at $Q^2 = 0$.

Minimum value of Q^2 achieved is **0.004** GeV^2

J. C. Bernauer *et al.*, Phys. Rev. Lett. **105**, 242001 (2010)

Proton radius measurement with ***pe*** *elastic scattering*

- E_p=100 MeV → Below the pion threshold for pp reactions.
- The maximum of the momentum transfer squared:

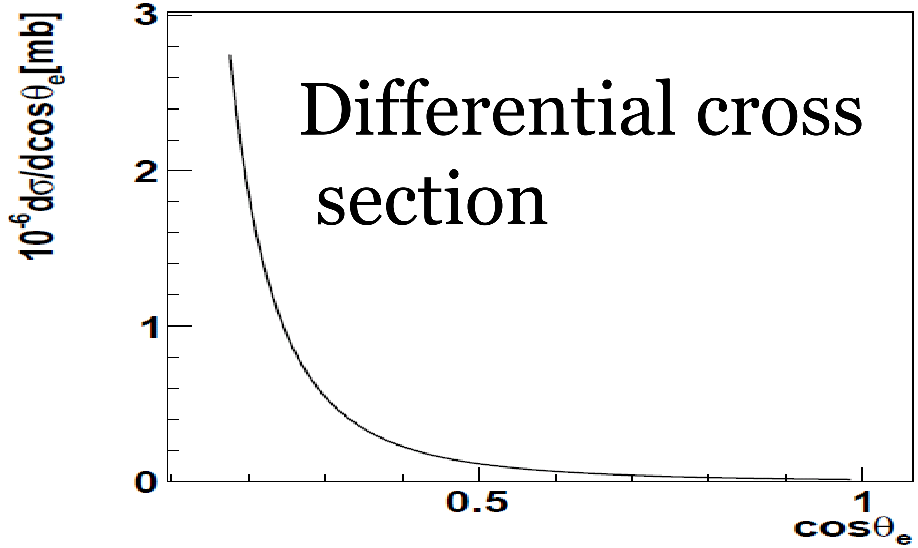


$$(Q^2)_{max} = \frac{4m^2(E^2 - M^2)}{M^2 + 2mE + m^2}$$

$$E_p = 100 \text{ MeV}, \ (Q^2)_{max} = 0.2 \times 10^{-6} \text{ GeV}^2.$$

Extension to low Q^2 to gives severe constraints to the fitting Procedure of the slope of G_E

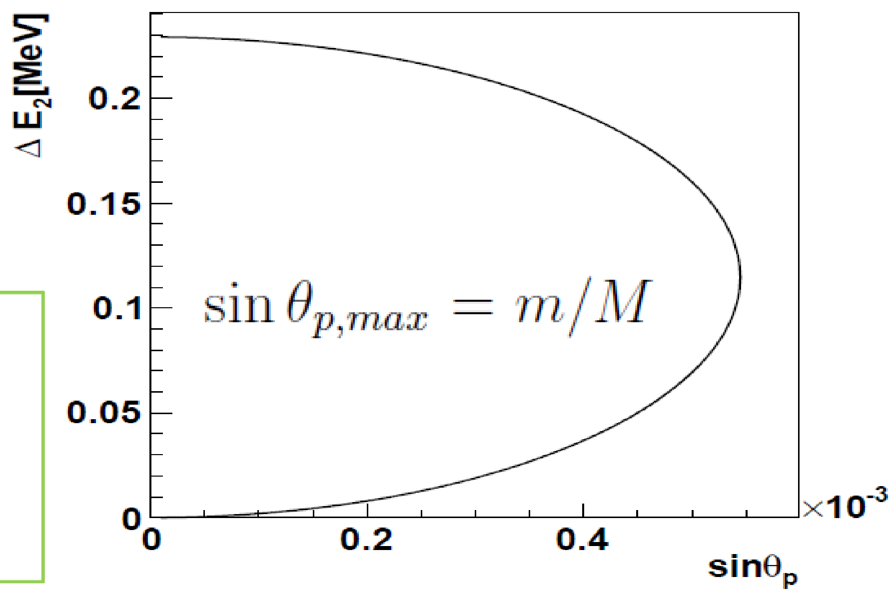
pe elastic scattering at E_p=100 MeV



$\mathcal{L}=10^{32}$ cm^{-2}s^{-1}

events=25 $\times 10^9$/s

$$\Delta E_2 = E_{scat.} - E_{beam}$$



Momentum resolution of the order 10^{-4} for the scattered protons is needed

Conclusion 1

Possibility to accessing low Q^2 values with high statistics in **pe** elastic scattering

→ precise measurement of r_c.

arXiv:1201.2572 [nucl-th].

Second application:

Polarized (anti)proton beams

(high energy application)

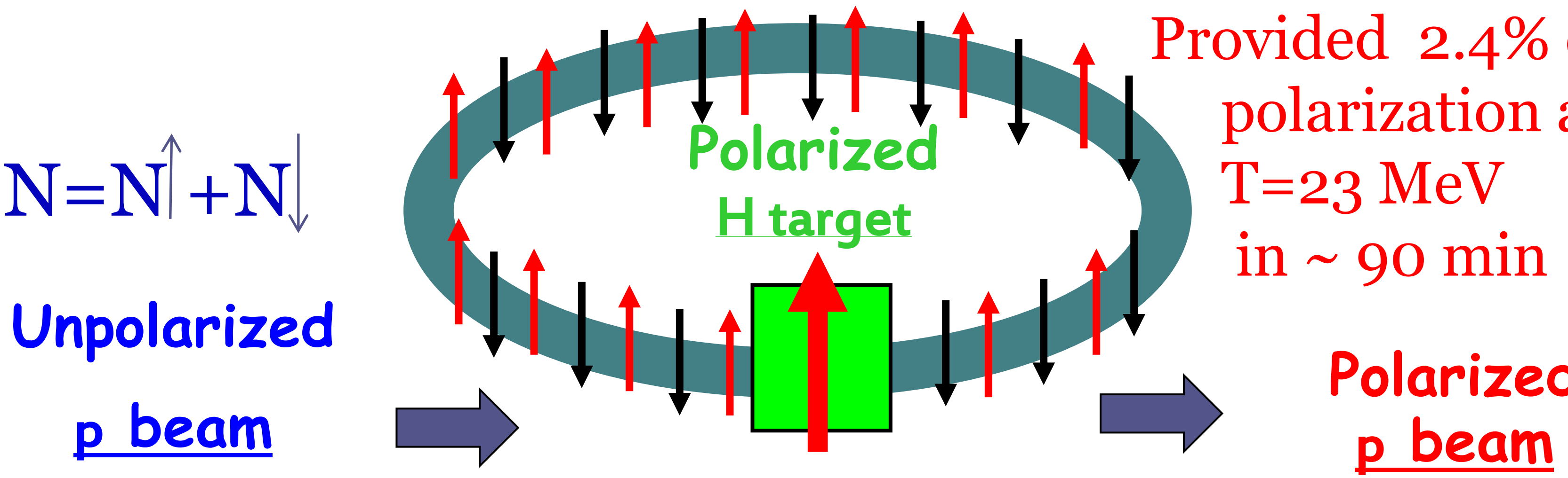
Polarized antiprotons: why?

- Knowledge of the the short range $\bar{p}p$ interaction (elastic scattering)
- Spin dependence of partonic processes
- Spin structure of the proton (annihilation into hadrons: pions, hyperons..)
- Transversity (Drell-Yan)
- Relative phase of proton electromagnetic form factors (annihilation into leptons)
- ……

(Reviews from J. Ellis, M. Anselmino, S. Brodsky, …)

Polarized (Anti)Proton Beam:

By repeated traversal of a beam through a polarized hydrogen target in a storage rings (Rathmann PRL 71 (1993))



Provided 2.4% of polarization at T=23 MeV in ~ 90 min

- Spin Filter: selective removal through **pp** scattering beyond the acceptance.
- Spin Flip: selective reversal the spin of the particle in one spin state.

Spin Transfer: from polarized electrons.

Polarization transfer coefficients:

$$p + \vec{e} \rightarrow \vec{p} + e$$

- Dirac density matrix:

$$u(p)\bar{u}(p) = (\hat{p} + m)\frac{1}{2}(1 - \gamma_5 \hat{s})$$

$$s_i^0 = \frac{\vec{p}_i.\vec{\chi}_i}{m_i}, \quad \vec{s}_i = \vec{\chi}_i + \frac{\vec{p}_i.\vec{\chi}_i\vec{p}_i}{m_i(E_i + m_i)}$$

$$|\mathcal{M}|^2 = 16\pi^2 \frac{\alpha^2}{k^4} L_{\mu\nu} W_{\mu\nu}$$

- Hadronic and leptonic tensors:

$$W_{\mu\nu} = J_\mu J_\nu^* = W_{\mu\nu}^0 + W_{\mu\nu}^1(s_{p1}) + W_{\mu\nu}^1(s_{p2}) + W_{\mu\nu}^2(s_{p1}, s_{p2})$$

$$L_{\mu\nu} = j_\mu j_\nu^* = L_{\mu\nu}^0 + L_{\mu\nu}^1(s_{e1}) + L_{\mu\nu}^1(s_{e2}) + L_{\mu\nu}^2(s_{e1}, s_{e2})$$

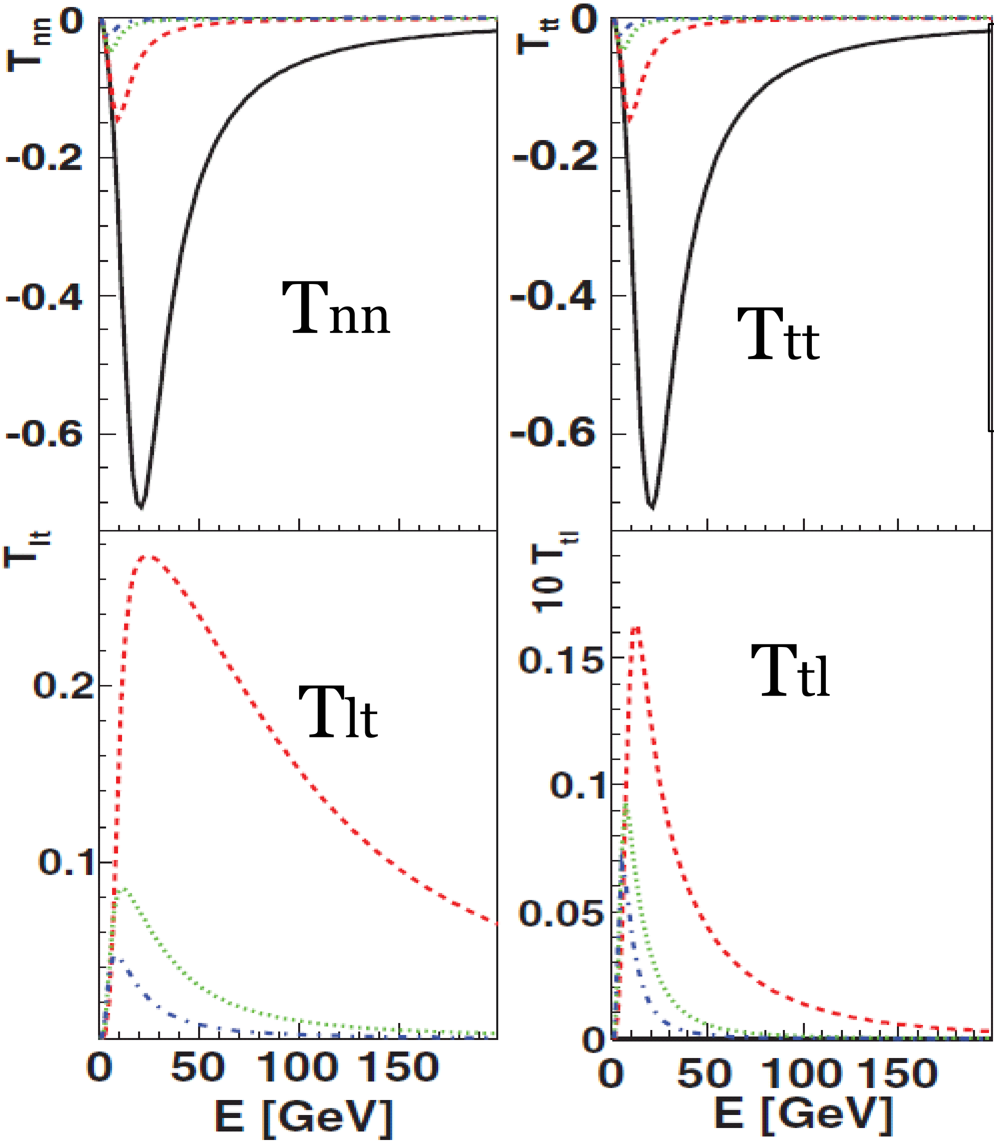
- Polarised cross section:

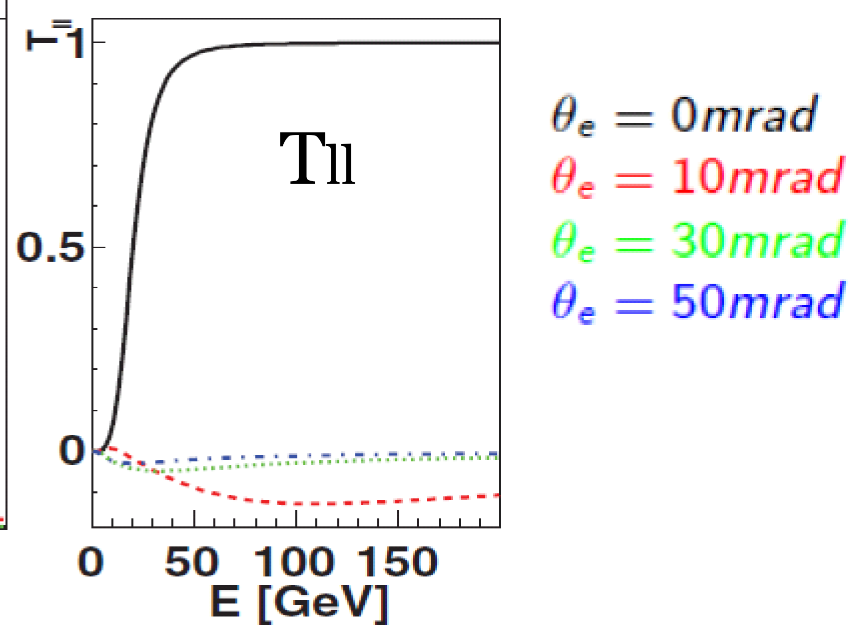
$$\frac{d\sigma}{dk^2} = \frac{d\sigma^{unp}}{dk^2}[1 + T_{\ell\ell}\chi_\ell^e\chi_\ell^p + T_{nn}\chi_n^e\chi_n^p + T_{tt}\chi_t^e\chi_t^p + T_{\ell t}\chi_\ell^e\chi_t^p + T_{t\ell}\chi_t^e\chi_\ell^p]$$

Polarization transfer coefficients

l (t)=longitudinal (transverse) along the incident proton beam,

n=normal wrt scatt. plane.

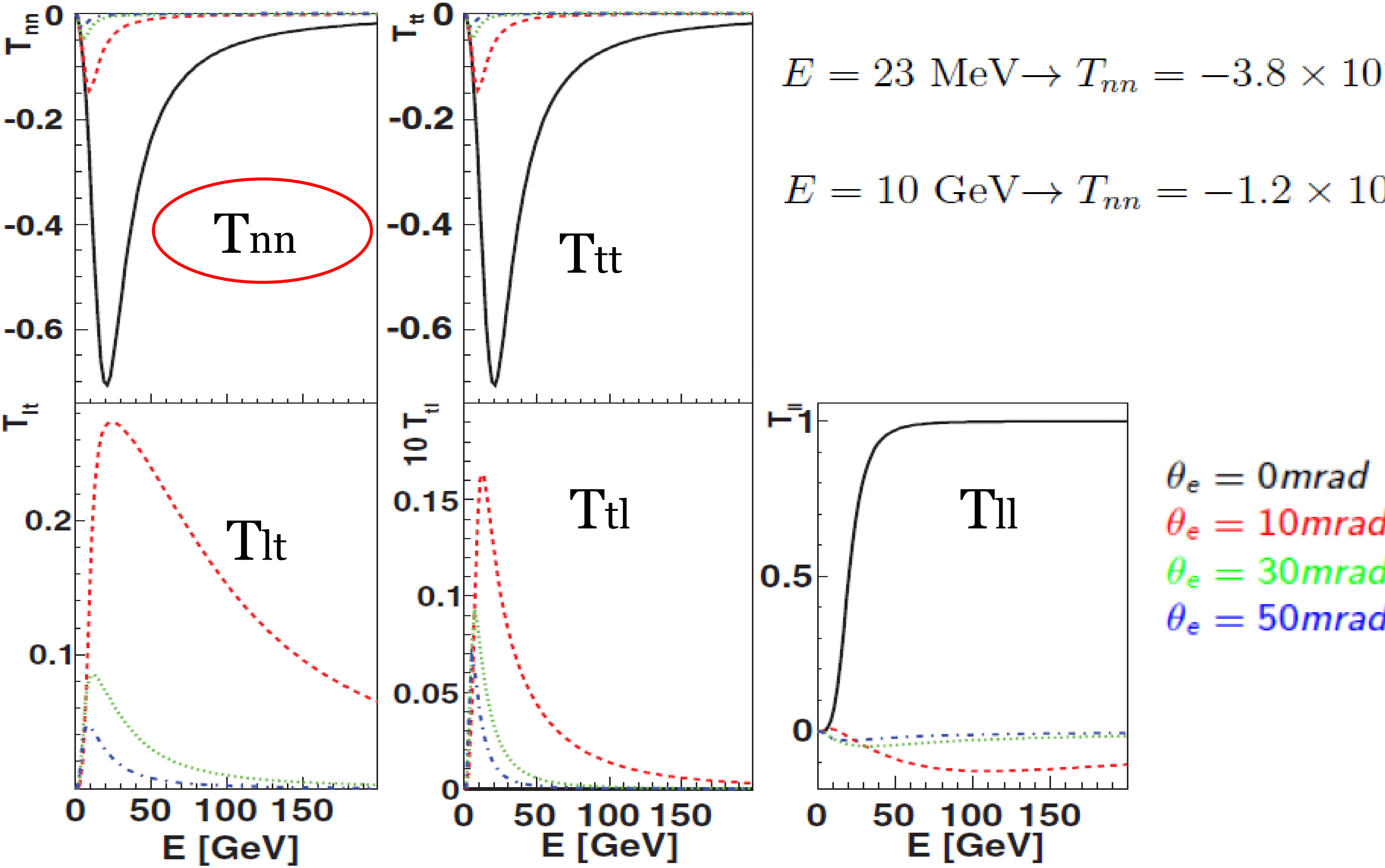




Polarization transfer coefficients

$$E = 23\ \mathrm{MeV} \rightarrow T_{nn} = -3.8 \times 10^{-12}$$

$$E = 10\ \mathrm{GeV} \rightarrow T_{nn} = -1.2 \times 10^{-6}$$



Conclusion 2

Large polarization effects appear in **pe** elastic scattering at energies between 10 GeV and 50 GeV.

Phys. Rev. C **84** (2011) 015212

Third application: polarimeters for high energy proton beams.

(I. V. Glavanakov *et al.*, Nucl. Instrum. Methodes Phys. Res. A 381, 275 (1996))

$$Angular\ asymmetry = C_{ij} P_i^{targ.} P_j^{beam}$$

Analyzing power reaction requirements:

1- Smallest theoretical uncertainties as possible at the level of process amplitude.

2- Large analyzing power C_{ij}.

$\vec{p}\vec{e}$ elastic scattering fulfills these requirements

Spin correlation coefficients (analyzing powers):

$$\vec{p} + \vec{e} \longrightarrow p + e$$

Hadronic and leptonic tensors:

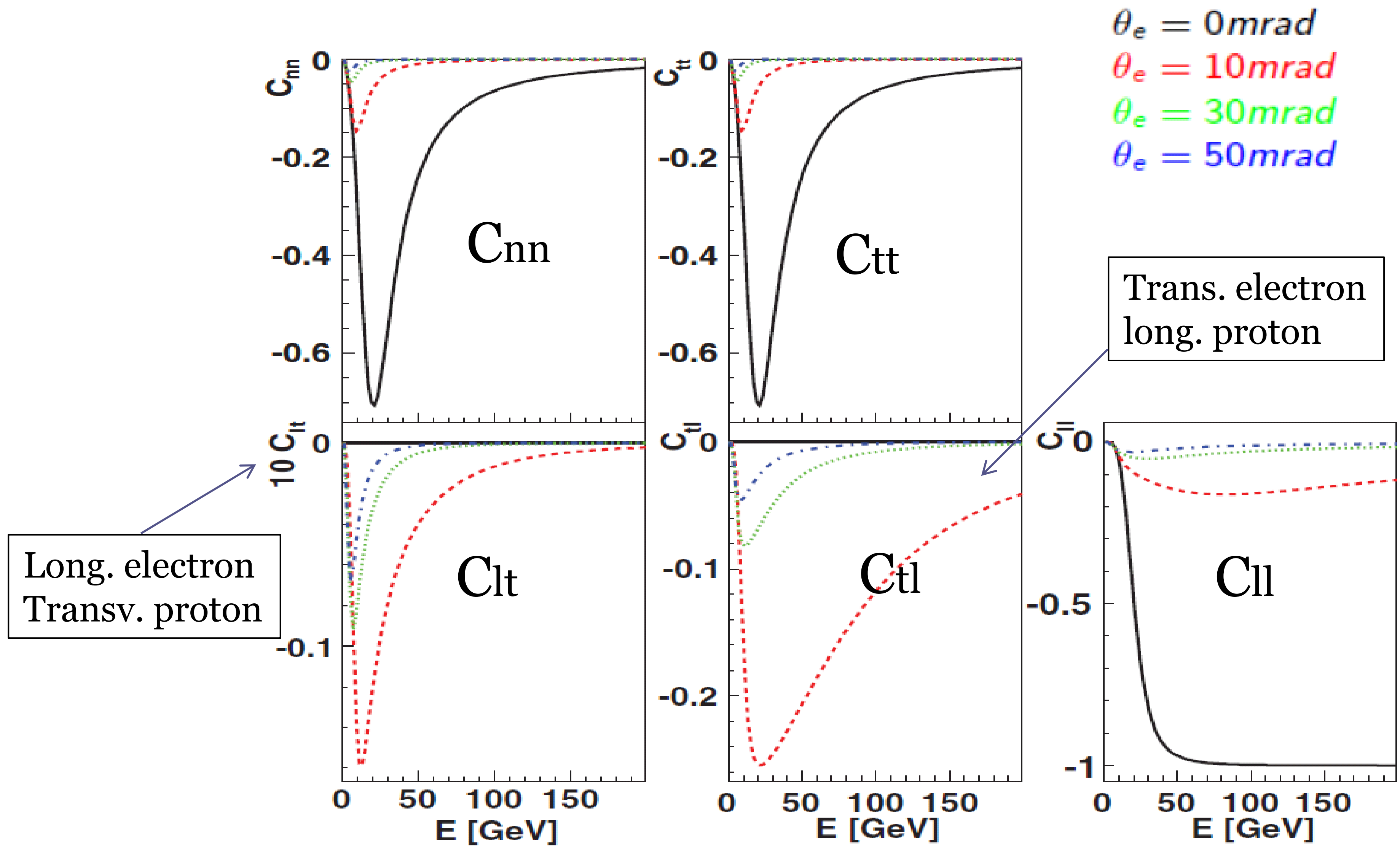
$$W_{\mu\nu} = J_\mu J_\nu^* = W_{\mu\nu}^0 + W_{\mu\nu}^1(s_{p1}) + W_{\mu\nu}^1(s_{p2}) + W_{\mu\nu}^2(s_{p1}, s_{p2})$$

$$L_{\mu\nu} = j_\mu j_\nu^* = L_{\mu\nu}^0 + L_{\mu\nu}^1(s_{e1}) + L_{\mu\nu}^1(s_{e2}) + L_{\mu\nu}^2(s_{e1}, s_{e2})$$

Polarized cross section:

$$\frac{d\sigma}{dk^2} = \frac{d\sigma^{unp}}{dk^2}[1 + C_{\ell\ell}\chi_\ell^e\chi_\ell^p + C_{nn}\chi_n^e\chi_n^p + C_{tt}\chi_t^e\chi_t^p + C_{\ell t}\chi_\ell^e\chi_t^p + C_{t\ell}\chi_t^e\chi_\ell^p]$$

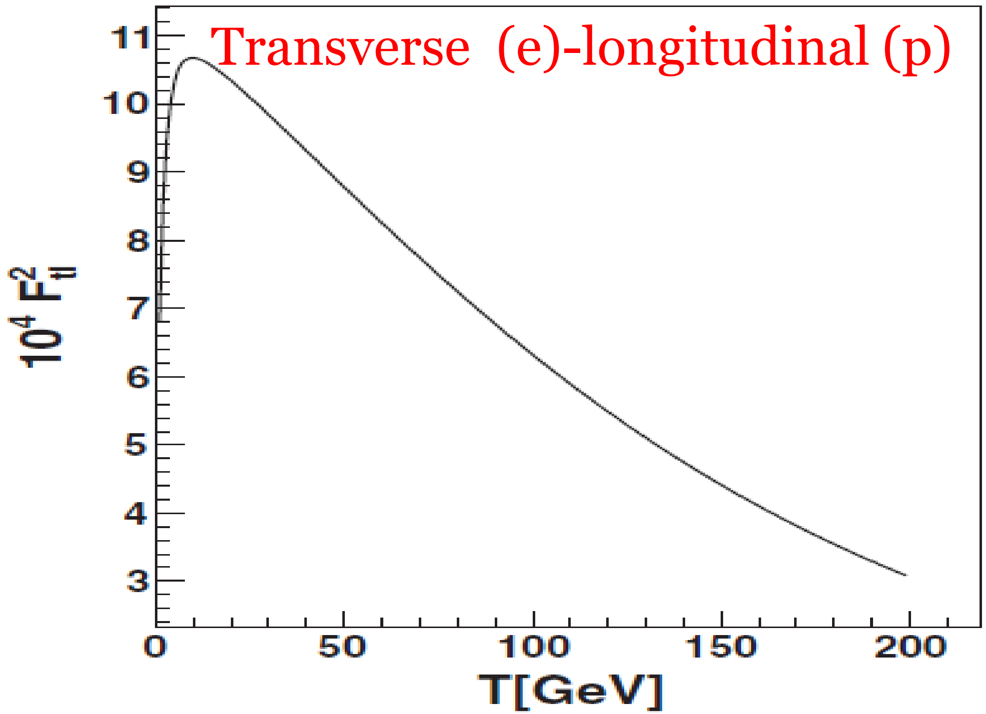
Spin correlation coefficients



The figure of merit

$$\left(\frac{\Delta P}{P}\right)^2 = \frac{2}{\mathcal{L} t_m d\sigma/d\Omega d\Omega C_{ij}^2 P^2}$$

$$F_{ij}^2 = \int \frac{d\sigma}{d\Omega} C_{ij}^2 d\Omega$$



At E~10 GeV,
$\mathcal{L}$ =10^{32} $cm^{-2}s^{-1}$
Δp=1% in t_m=3min

Conclusions

Relativistic description of proton-electron scattering: kinematics, differential cross section and polarization phenomena.

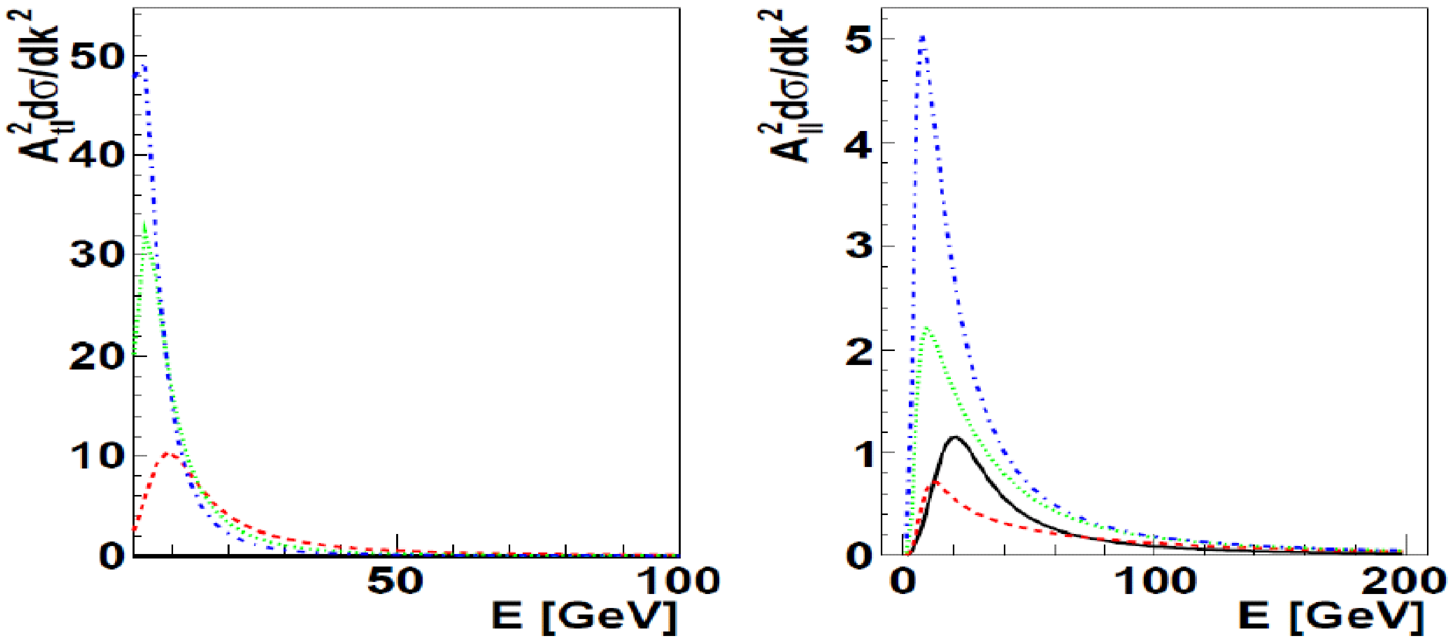
- Possibility to accessing low Q^2 values with high statistics→ precise measurement of r_c.
- Polarization effects are large at energies in the GeV range:
  Possible applications to polarized physics for high energy (anti)proton beams.

Thank you for your attention

The figure of merit

$$\mathcal{F}^2(\theta_p) = \epsilon(\theta_p) A_{ij}^2(\theta_p), \quad \epsilon(\theta_p) = N_f(\theta_p)/N_i$$

$$\left(\frac{\Delta P(\theta_p)}{P}\right)^2 = \frac{2}{N_i(\theta_p)\mathcal{F}^2(\theta_p)P^2} = \frac{2}{L t_m (d\sigma/d\Omega) d\Omega A_{ii}^2(\theta_p) P^2}$$



At E~10 GeV, $L = 10^{32}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$, *Δp=1% in t=3 min*

Polarized antiprotons: how?

- Parity-violating (in flight) decay of anti-Λ^0 hyperons
  P=45%, I(p)~ 10^4 s^{-1}

(FermiLab, A. Bravar, PRL 77 (1996),
D.P. Grosnick, PRC 55,1159 (1997), NIMA290(1990))

- Stern-Gerlach separation in an inhomogeneous magnetic field (too expensive)

- Elastic scattering on C, LH2…

and also….

Methods for measuring proton charge radius

- Hydrogen spectroscopy (CODATA, Lamb shift)

○ **Dirac →** Energy levels of hydrogen electron depend only on the principal quantum number《n》.

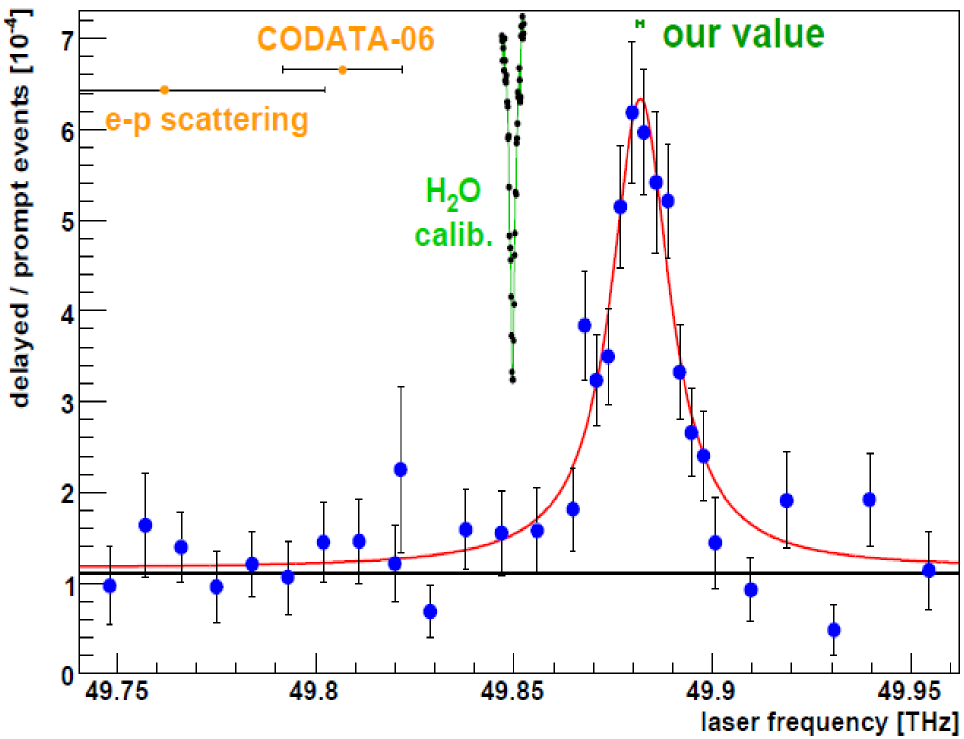
○ Proton structure corrections (at leading order):

$$\Delta E^{FS} = \frac{2(Z\alpha)^4}{3n^3} m_r^3 \, r_{\mathrm{p}}^2 \, \delta_{l0}$$

○ Other QED effects to the Lamb shift:
self energy, vacuum polarization, nuclear motion …

Methods for measuring proton charge radius

- Muonic hydrogen spectroscopy (CODATA, Lamb shift)



PSI Experiment

Nature **466**, 213-216 (8 July 2010)

X-ray timing and 2S $_{1/2}$ - 2P $_{3/2}$ transition spectra

Methods for measuring proton charge radius

- Hydrogen spectroscopy (CODATA, Lamb shift)

- Muonic Hydrogen spectroscopy (Lamb shift)

- Elastic e−p scattering to determine electric form factor:

$$< r_c^2 >= -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$