From SL to TL form factors in the Skyrme soliton model

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Tesi di Laurea

FATTORI DI FORMA ELETTROMAGNETICI DEL NUCLEONE NELLA REGIONE TEMPO

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- General remarks on TLFFs
- Skyrme model
- Boosting
- Skyrme model with vector mesons
- Analytic continuation from SL to TL

The electric and magnetic space-like form factor can be computed from the currents. In the Breit frame:

$$G_E(-\mathbf{q}^2)\langle s_3' | s_3 \rangle = \left\langle N'\left(\frac{\mathbf{q}}{2}\right) \middle| J^0(0) \middle| N\left(-\frac{\mathbf{q}}{2}\right) \right\rangle$$

$$-\frac{i}{2M} G_M(-\mathbf{q}^2)\langle s_3' | \varepsilon^{ijk} q_j \sigma_k | s_3 \rangle = \left\langle N'\left(\frac{\mathbf{q}}{2}\right) \middle| J^i(0) \middle| N\left(-\frac{\mathbf{q}}{2}\right) \right\rangle$$

Crossing symmetry + analytic continuation provide support to the idea that there is a unique function describing both SL and TL form factors.

In the SL region FFs are Fourier transform of the electromagnetic currents; the analitic continuation in the TL region (from $Q^2 > 0$ to $Q^2 < 0$) should provide the TLFFs.

Simple models, as e.g. MIT bag model cannot be used since they do not describe the hadronization process:

- The current vanishes for $r > R$ and therefore no singularities are possible at finite Q^2
- For $Q^2 \to +$ infinity there is no singularity (Fourier transform)
- All analytic functions are singular at finite or at infinity
- the TLFF would be singular for $Q^2 \to -$ infinity

Analytic properties of FFs

- $F(z)$ real and not singular for $z < t_0$ where
  - $t_0 = 4\ m_\pi^2$ for isoscalar
  - $t_0 = 9\ m_\pi^2$ for isovector
- $F(z)$ has a cut for arg $z = 0$ and $z >= t_0$

Skyrme model

$$\mathcal{L} = -\frac{f_\pi^2}{4}\mathrm{Tr}(D_\mu U^\dagger D^\mu U) + \frac{1}{32g^2}\mathrm{Tr}[U^\dagger D_\mu U, U^\dagger D_\nu U]^2 + \mathcal{L}_{WZ} + \mathcal{L}_M$$

$$\mathcal{L}_{\mathrm{WZ}} = \frac{N_c}{48\pi^2}\varepsilon^{\mu\nu\lambda\sigma}\,\mathrm{Tr}[e A_\mu \hat{Q}(U^\dagger \partial_\nu U\, \partial_\lambda U^\dagger\, \partial_\sigma U + \partial_\nu U\, \partial_\lambda U^\dagger\, \partial_\sigma U\, U^\dagger)]$$

$$\mathcal{L}_{\mathrm{WZ}} = -\frac{e}{2}A_\mu B^\mu \qquad B^\mu = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\lambda\sigma}\,\mathrm{Tr}(U^\dagger \partial_\nu U\, \partial_\lambda U^\dagger\, \partial_\sigma U)$$

$$\mathcal{L}_M = \frac{f_\pi^2}{2}m_\pi^2\,\mathrm{Tr}U$$

$$U \equiv \exp\left(\frac{i}{f_\pi}\boldsymbol{\pi}\cdot\boldsymbol{\tau}\right)$$

Hedgehog ansatz

$$\boldsymbol{\pi} = f_\pi F(r)\hat{\mathbf{r}} \qquad U_S = \exp[\, i F(r)\hat{\mathbf{r}}\cdot\boldsymbol{\tau}]$$

$$F'' = -\frac{2}{r}F' + \frac{\operatorname{sen} 2F}{r^2} - \frac{8g^2}{f_\pi^2}\left(\frac{\operatorname{sen} 2F \operatorname{sen}^2 F}{r^4} - \frac{F' \operatorname{sen} 2F}{r^2} - \frac{F'' \operatorname{sen}^2 F}{r^2}\right)$$

$$F(0) = \pi \qquad F(\infty) = 0$$

$$F(r) \overset{r\to\infty}{\sim} \frac{a_1 e^{-m_\pi r}}{r} + \frac{a_2 e^{-m_\pi r}}{r^2}$$

Non-relativistic quantization of the skyrmion

$$U(\mathbf{r}, t) = A(t) U_S (\mathbf{r} - \mathbf{X}(t)) A^\dagger(t)$$

$$L = -M + \frac{1}{2} M \dot{\mathbf{X}}^2 + \Lambda \,\mathrm{Tr}(\dot{A}^\dagger \dot{A})$$

$$\mathbf{P} = \frac{\partial L}{\partial \dot{\mathbf{X}}} = M \dot{\mathbf{X}}$$

$$[X_i, P_j] = i \delta_{ij}$$

$$\mathbf{S} = -i \,\mathrm{Tr}(\boldsymbol{\tau} A^\dagger \dot{A})$$

$$[S_i, S_j] = i \varepsilon_{ijk} S^k$$

$$H = M + \frac{1}{2M} \mathbf{P}^2 + \frac{1}{2\Lambda} \mathbf{S}^2$$

Currents in a $1/N_c$ expansion

P is always associated with M and S is associated with Λ
M and Λ both scale as N_c

$$J_0^S(0) = b_0(\mathbf{X}^2) + O(1/N_c)$$

$$J_i^S(0) = \frac{1}{\Lambda} b_1(\mathbf{X}^2)\varepsilon_{ijk} X^j S^k + \frac{1}{2M}\left\{P^j, b_2(\mathbf{X}^2)\delta_{ij} + b_3(\mathbf{X}^2)\frac{X_i X_j}{\mathbf{X}^2}\right\}$$
$$+ O(1/N_c^2);$$

$$J_{03}^V(0) = -\frac{1}{2\Lambda}[t_1(\mathbf{X}^2)\mathbf{X}^2\delta_{ij} - t_2(\mathbf{X}^2) X_i X_j]\{S^i, I_3^j\}$$
$$-\frac{1}{2M}\varepsilon_{ijk}\{P^i, t_3(\mathbf{X}^2) X^j\} I_3^k + O(1/N_c^2);$$

$$J_{i3}^V(0) = -t_0(\mathbf{X}^2)\varepsilon_{ijk} X^j I_3^k + O(1/N_c)$$

Currents in the hedgehog ansatz

$$b(\mathbf{X}^2) \equiv b_i(\mathbf{X}^2) = -\frac{1}{4\pi^2}\frac{\operatorname{sen}^2 F}{\mathbf{X}^2}F' \qquad (i = 0, 1, 2) \qquad b_3(\mathbf{X}^2) = 0$$

$$t(\mathbf{X}^2) \equiv t_i(\mathbf{X}^2) = f_\pi^2\frac{\operatorname{sen}^2 F}{\mathbf{X}^2} + \frac{1}{g^2}\frac{\operatorname{sen}^2 F}{\mathbf{X}^2}\left(F'^2 + \frac{\operatorname{sen}^2 F}{\mathbf{X}^2}\right)$$

$$J_0^S(0) = b(\mathbf{X}^2) + O(1/N_c)$$

$$J_i^S(0) = \frac{1}{\Lambda}b(\mathbf{X}^2)\varepsilon_{ijk}X^j S^k + \frac{1}{2M}\{P^j, b(\mathbf{X}^2)\delta_{ij}\} + O(1/N_c^2)$$

$$J_{03}^V(0) = -\frac{1}{2\Lambda}t(\mathbf{X}^2)(\mathbf{X}^2\delta_{ij} - X_i X_j)\{S^i, I_3^j\}$$

$$-\frac{1}{2M}\varepsilon_{ijk}\{P^i, t(\mathbf{X}^2)X^j\}I_3^k + O(1/N_c^2);$$

$$J_{i3}^V(0) = -t(\mathbf{X}^2)\varepsilon_{ijk}X^j I_3^k + O(1/N_c)$$

SLFFs in the non-relativistic Skyrme model

$$G_E^S(Q^2) = \int d^3\mathbf{r}\, j_0(Qr)\, b(r^2)$$ $O(N_c^0)$

$$G_E^V(Q^2) = \pm\int d^3\mathbf{r}\, j_0(Qr)\frac{1}{3\Lambda} r^2\, t(r^2)$$ $O(N_c^{-1})$

$$G_M^S(Q^2) = \frac{2M}{Q}\int d^3\mathbf{r}\, j_1(Qr)\frac{1}{2\Lambda} r\, b(r^2)$$ $O(N_c^0)$

$$G_M^V(Q^2) = \pm\frac{2M}{Q}\int d^3\mathbf{r}\, j_1(Qr)\frac{1}{3} r\, t(r^2)$$ $O(N_c^1)$

$$Q = +\sqrt{Q^2}$$

SLFFs from Braaten, Tse, Willcox, PRL 56 (1986) 2011

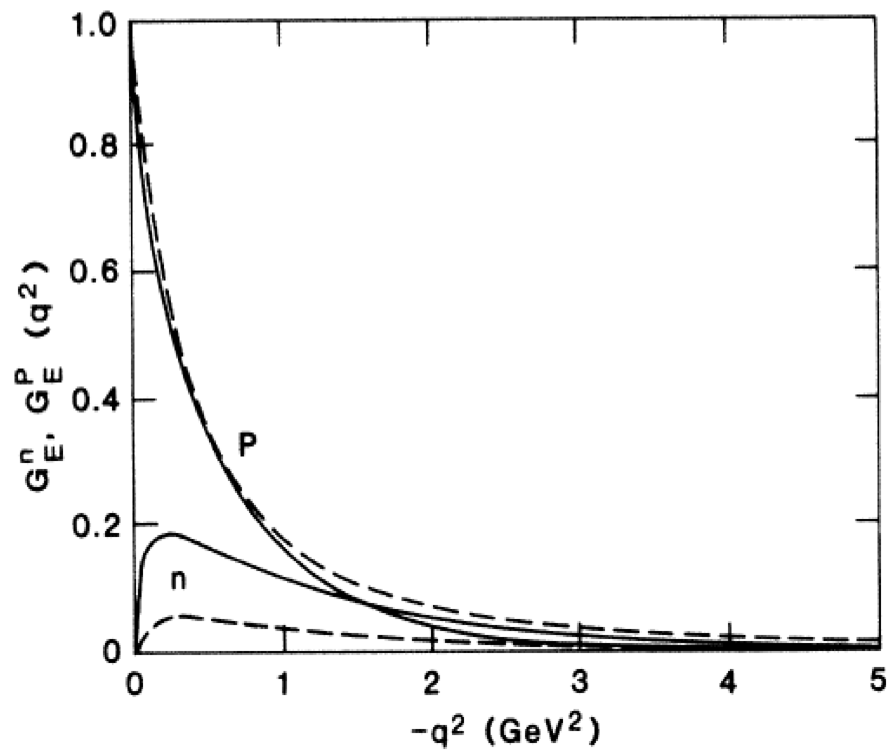

FIG. 1. Proton and neutron electric form factors: Skyrme-model predictions (solid lines) and dipole parametrizations (dashed lines).

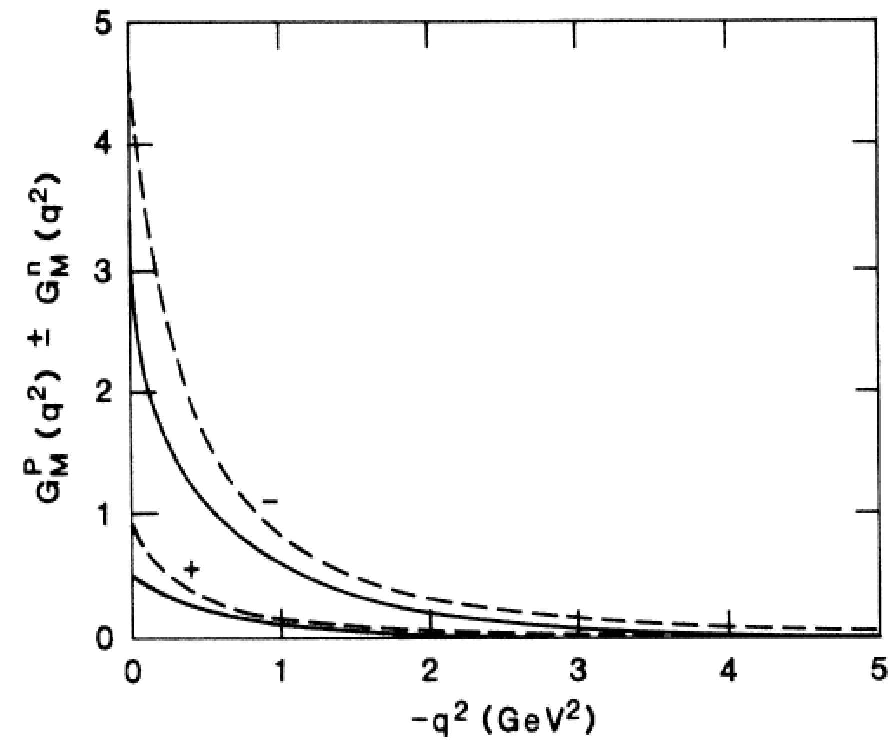

FIG. 2. Nucleon isoscalar and isovector magnetic form factors: Skyrme-model predictions (solid lines) and dipole parametrizations (dashed lines).

Boosting the soliton

Ji, PLB 254 (1991) 456

Main idea is to quantize also large momenta

$(t, \mathbf{x})$ Are the coordinates for the frame in which the soliton moves with velocity v

$$r_i = \left(\delta_{ij} + \frac{\gamma - 1}{v^2} v_i v_j \right) (x^j - v^j t) \equiv W_{ij} (x^j - v^j t)$$

$$\tau = \gamma (t - \mathbf{x} \cdot \mathbf{v}) \qquad \gamma = \frac{1}{\sqrt{1 - v^2}}$$

The relation between the soliton in the rest and in the boosted frame reads:

$$U_S(t, \mathbf{x}) = U_S(\mathbf{r}) = U_S(W(\mathbf{x} - \mathbf{v}t))$$

The lagrangian in the rest and in the boosted frame reads:

$$L_{LAB} = -M \qquad L_{BREIT} = \sqrt{1 - v^2} L_{LAB} = -\sqrt{1 - v^2} M$$

Quantization is done by introducing the center of soliton momentum

$$\mathbf{P} = \frac{\partial L_{BREIT}}{\partial \dot{\mathbf{R}}} = M \gamma \mathbf{v} \qquad \text{where} \qquad \mathbf{R} \equiv \mathbf{v} t$$

Vibrational degrees of freedom are neglected (heavy soliton), but rotational degrees of freedom are considered by introducing the Pauli-Lubanski operator

$$S_{\mu} = -\frac{1}{4}\varepsilon_{\mu\nu\alpha\beta}\,\sigma^{\nu\alpha}\,P^{\beta}$$

Currents are evaluated in the relativistically moving and rotating frame

$$J_0^S(0) = \frac{1}{\Lambda} b(\mathbf{R}^2)\,\varepsilon_{ijk}\,R^j\,S^k\,\frac{P^i}{M} + b(\mathbf{R}^2)\,\frac{P_0}{M}$$

$$J_i^S(0) = \frac{1}{\Lambda} b(\mathbf{R}^2)\,\varepsilon_{ljk}\,R^j\,S^k\,W_i^l + \frac{1}{2M}\{P_i\,,\,b(\mathbf{R}^2)\}$$

$$J_{03}^V(0) = -t(\mathbf{R}^2)\left[\varepsilon_{ijk}\,R^j\,I_3^k\,\frac{P^i}{M} + \frac{1}{2\Lambda}(\mathbf{R}^2\delta_{ij} - R_i\,R_j)\{S^i\,,\,I_3^j\}\right]$$

$$J_{i3}^V(0) = -t(\mathbf{R}^2)\left[\varepsilon_{ljk}\,R^j\,I_3^k\,W_i^l + \frac{1}{2\Lambda}(\mathbf{R}^2\delta_{ij} - R_i\,R_j)\{S^i\,,\,I_3^j\}\right]$$

Comparing relativistic and non-relativistic SLFFs

$$G_E^S(-\mathbf{q}^2) = \int d^3\mathbf{r}\; j_0\left(\frac{|\mathbf{q}|}{\gamma} r\right) b(\mathbf{r}^2)$$

$$G_E^S\Big|_{NR}(-\mathbf{q}^2) = \int d^3\mathbf{r}\; j_0(|\mathbf{q}|r)\, b(\mathbf{r}^2)$$

$$G_M^S(-\mathbf{q}^2) = \frac{1}{\gamma^2}\frac{2M\gamma}{|\mathbf{q}|}\int d^3\mathbf{r}\; j_1\left(\frac{|\mathbf{q}|}{\gamma} r\right)\frac{1}{2\Lambda} r\, b(r^2)$$

$$G_M^S\Big|_{NR}(-\mathbf{q}^2) = \frac{2M}{|\mathbf{q}|}\int d^3\mathbf{r}\; j_1(|\mathbf{q}|r)\frac{1}{2\Lambda} r\, b(r^2)$$

From velocity to 4-momentum in SL

$$\begin{cases} Q^2 = \mathbf{q}^2 \\ \gamma^2 = 1 + \dfrac{Q^2}{4M^2} \end{cases}$$

$$G_E(Q^2) = G_E\big|_{NR}\left(\frac{Q^2}{1+\frac{Q^2}{4M^2}}\right) \qquad G_M(Q^2) = \frac{1}{1+\frac{Q^2}{4M^2}} G_M\big|_{NR}\left(\frac{Q^2}{1+\frac{Q^2}{4M^2}}\right)$$

Notice that G_E(+ infinity) = $G_{E\,|NR}$ (4 M^2),
maybe due to the absence of vibration (rigid soliton)
$G_M \rightarrow 1/Q^2$

SLFFs with the relativistic quantization

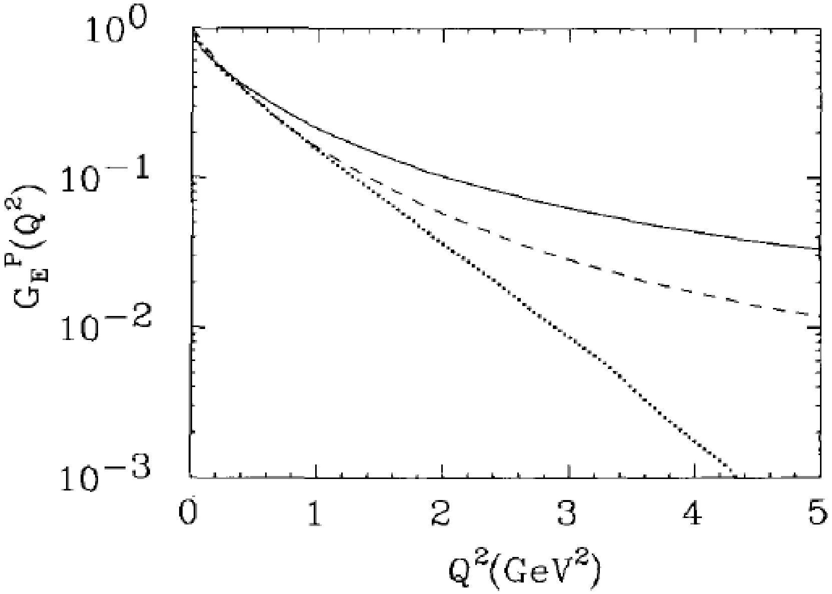

Fig. 1. The proton's electric form factor. The dotted curve is the non-relativistic Skyrme model calculation from ref. [8]. The full curve is the relativistic calculation. The dashed curve is a fit to experimental data in ref. [11].

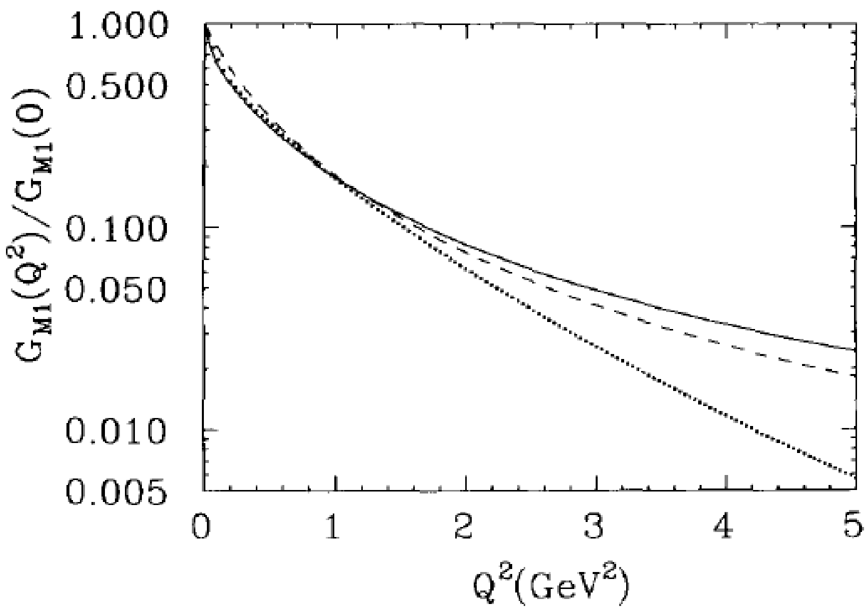

Fig. 2. The nucleon's isovector magnetic form factor. Different curves have the same meanings as in fig. 1.

From velocity to 4-momentum in TL

$$\begin{cases} Q^2 = -\mathbf{q}^2 - 4M^2 \\ \gamma^2 = -\dfrac{Q^2}{4M^2} \end{cases}$$

No boost at threshold!

$$G_E(Q^2) = G_E\big|_{NR}\left[-4M^2\left(2 + \frac{4M^2}{Q^2}\right)\right]$$

$$G_M(Q^2) = -\frac{4M^2}{Q^2} G_M\big|_{NR}\left[-4M^2\left(2 + \frac{4M^2}{Q^2}\right)\right]$$

SLFFs and TLFFs have the same asymptotic behaviour!

(no special reason why electric and magnetic TLFFs should be equal at threshold…)

Skyrme model with vector mesons

Meissner, Keiser, Wirzba, Weise

ρ and w are gauge bosons of a hidden symmetry SU(2) x U(1)

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(D_\mu U\, D^\mu U^\dagger) - \frac{f_\pi^2}{2}\mathrm{Tr}[D_\mu U^{1/2}(U^\dagger)^{1/2} + D_\mu (U^\dagger)^{1/2} U^{1/2}]^2$$
$$- \frac{1}{2g^2}\mathrm{Tr}(F_{\mu\nu} F^{\mu\nu}) + \mathcal{L}_{WZ} + \mathcal{L}_M \,.$$

$$D_\mu = \partial_\mu - \frac{i\,g}{2}(\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu + \omega_\mu)$$

Generalizing the hedgehog ansatz

$$\omega_\mu(\mathbf{r}) = \omega(r)\,\delta_{\mu 0} \qquad \rho_{0a}(\mathbf{r}) = 0 \qquad \rho_{ia}(\mathbf{r}) = \frac{1}{g}\varepsilon_{ija}\,\hat{r}^j\, G(r)$$

Field equations

$$F'' = -\frac{2}{r}F' + \frac{1}{r^2}[4(G+1)\operatorname{sen} F - \operatorname{sen} 2F] + m_\pi^2 \operatorname{sen} F$$
$$+ \frac{3g\,\omega'}{8\pi^2\, f_\pi^2\, r^2}[-2\operatorname{sen}^2 F + G\,(G+2)\cos 2F]\,;$$

$$G'' = 2g^2\, f_\pi^2\,(G+1-\cos F)F' + \frac{1}{r^2}G\,(G+1)\,(G+2) + \frac{3g^3}{16\pi^2}\omega'\,(G+1)\operatorname{sen} 2F$$

$$\omega'' = -\frac{2}{r}\omega' + 2g^2\, f_\pi^2\,\omega - \frac{3g}{4\pi^2\, r^2}F'\operatorname{sen}^2 F$$
$$+ \frac{3g}{8\pi^2\, r^2}[G\,(G+2)\,F'\cos 2F + G'\,(G+1)\operatorname{sen} 2F]\,.$$

$$G(0) = -2 \qquad G(\infty) = 0 \qquad \omega'(0) = \omega(\infty) = 0$$

Asymptotic behaviour of the fields

$$F(r) \overset{r\to 0}{\sim} \pi + a_1 r + o(r^2)$$

$$F(r) \overset{r\to\infty}{\sim} e^{-m_\pi r}\left(\frac{a_5}{r} + \frac{a_6}{r^2}\right)$$

$$G(r) \overset{r\to 0}{\sim} -2 + a_2 r^2 + o(r^3)$$

$$G(r) \overset{r\to\infty}{\sim} \frac{a_7 e^{-2m_\pi r}}{r^2}$$

$$\omega(r) \overset{r\to 0}{\sim} a_3 + a_4 r^2 + o(r^3)$$

$$\omega(r) \overset{r\to\infty}{\sim} \frac{a_8 e^{-3m_\pi r}}{r^5}$$

Non-relativistic quantization of the skyrmion with vector mesons

$$U(\mathbf{r},\, t) = A(t) U_S (\mathbf{r} - \mathbf{X}(t))\, A^\dagger (t)$$

For a time-dependent solution there are 3 more field components

$$\boldsymbol{\tau} \cdot \boldsymbol{\rho}_0 (\mathbf{r},\, t) = \frac{2}{g} A(t)\; \boldsymbol{\tau} \cdot [\xi_1(r) \mathbf{K} + \xi_2(r) (\mathbf{K} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}]\, A^\dagger (t)$$

$$\omega_i (\mathbf{r},\, t) = \frac{\Phi(r)}{r} \varepsilon_{i\,j\,k}\, K^j\, \hat{r}^k$$

Here **K** is the rotational frequency

Field equations for the new components

$$\xi_1'' = -\frac{2}{r}\xi_1' - \frac{3g^3}{32\pi^2 r^2}\Phi'(G+1)\,\mathrm{sen}\,2F - 4 f_\pi^2 g^2\,\mathrm{sen}^2\frac{F}{2}$$

$$+\frac{1}{r^2}[G^2(\xi_1 - 1) - 2(G+1)\xi_2] + 2 f_\pi^2 g^2 \xi_1 \;;$$

$$\xi_2'' = -\frac{2}{r}\xi_2' - \frac{3g^3}{32\pi^2 r^2}[\Phi F' + \frac{1}{4}\Phi'(G+1)\,\mathrm{sen}\,2F] + 4 f_\pi^2 g^2\,\mathrm{sen}^2\left(\frac{F}{2}\right)$$

$$+\frac{1}{r^2}[G^2(\xi_1 - \xi_2 - 1) + 3(G^2 + 2G + 2)\xi_2] + 2 f_\pi^2 g^2 \xi_2 \;;$$

$$\Phi'' = \frac{2}{r^2}\xi_1' - \frac{3g}{4\pi^2}F'(2\,\mathrm{sen}^2 F - \xi_1 - \xi_2)$$

$$+\frac{3g}{8\pi^2}[2F'\cos 2F\,(G - G\xi_1 - \xi_1) + (G' - G'\xi_1 - G\xi_1' - \xi')\,\mathrm{sen}\,2F] + 2 f_\pi^2 g^2 \Phi \,.$$

$$\xi'_{1,\,2}(0) = \xi_{1,\,2}(\infty) = 0 \qquad \xi_{1,\,2}(r) \overset{r\to\infty}{\sim} \frac{a_4' \, e^{-2m_\pi r}}{r^2}$$

$$\Phi(0) = \Phi(\infty) = 0 \qquad \Phi(r) \overset{r\to\infty}{\sim} \frac{a_5' \, e^{-3m_\pi r}}{r^3}$$

Currents in Skyrme model with vector mesons

$$J_0^S = -\frac{m_\omega^2}{3g}\omega$$

$$J_i^S = -\frac{m_\omega^2}{12\lambda g}\Phi\frac{1}{r^2}\varepsilon_{ijk}\sigma^j r^k$$

$$J_0^V = \frac{f_\pi^2}{3\lambda}\left[4\,\mathrm{sen}^4\left(\frac{F}{2}\right)+(1+2\cos F)\xi_1+\xi_1\right]$$

$$J_i^V = \frac{2f_\pi^2}{3}\left[2\,\mathrm{sen}^4\left(\frac{F}{2}\right)-G\cos F\right]\frac{1}{r^2}\varepsilon_{ijk}\sigma^j r^k$$

SLFFs with vector mesons

$$G_E^S(Q^2) = \int d^3\mathbf{r}\; j_0(Qr)\left(-\frac{m_\omega^2}{3g}\omega\right)$$

$$G_E^V(Q^2) = \pm\int d^3\mathbf{r}\; j_0(Qr)\frac{f_\pi^2}{3\lambda}\left[4\,\mathrm{sen}^4\left(\frac{F}{2}\right) + (1+2\cos F)\xi_1 + \xi_2\right]$$

$$G_M^S(Q^2) = \frac{2M}{Q}\int d^3\mathbf{r}\; j_1(Qr)\left(-\frac{m_\omega^2}{12\lambda g}\Phi\frac{1}{r}\right)$$

$$G_M^V(Q^2) = \pm\frac{2M}{Q}\int d^3\mathbf{r}\; j_1(Qr)\frac{2f_\pi^2}{3}\left[2\,\mathrm{sen}^4\left(\frac{F}{2}\right) - G\cos F\right]\frac{1}{r}$$

From SL to TL

Strategy:

- parametrize the currents, taking into account the asymptotic behaviours so that they are exact in the parametrization.
- Evaluate **analytically** the FFs in the SL region
- Use the same analytic expression of the FFs in both SL and TL region

$$G_E^S(Q^2) = \int d^3\mathbf{r}\; j_0(Q\,r)\left[-\frac{m_\omega^2}{3g}\,\omega(r)\right]$$

$$-\frac{m_\omega^2}{3g}\,\omega(r) = \omega_a(r) + \omega_b(r)$$

$$\omega_a(r) = \frac{\omega_{a0}\, e^{-3m_\pi r}}{(r^2+\omega_{a1}^2)(r^2+\omega_{a2}^2)(r^2+\omega_{a3}^2)} \qquad \omega_b(r) = \frac{\omega_{b0}\, r\, e^{-3m_\pi r}}{(r^2+\omega_{b1}^2)(r^2+\omega_{b2}^2)(r^2+\omega_{b3}^2)}$$

$$G_E^S(Q^2) = \int_0^\infty 4\pi r^2\, dr \frac{e^{iQr} - e^{-iQr}}{2iQr}[\omega_a(r) + \omega_b(r)]$$

$$= G_{Ea}^S(Q^2) + G_{Eb}^S(Q^2).$$

$$\omega_a(r) = \omega_{a0}\, e^{-3m_\pi r} \sum_{j=1}^{3} \frac{\Omega_{aj}}{r^2 + \omega_{aj}^2}$$

$$= \omega_{a0}\, e^{-3m_\pi r} \sum_{j=1}^{3} \frac{\Omega_{aj}}{2ib_{aj}} \left(\frac{1}{r - i\omega_{aj}} - \frac{1}{r + i\omega_{aj}} \right).$$

$$\omega_b(r) = \omega_{b0}\, e^{-3m_\pi r} \sum_{j=1}^{3} \frac{\Omega_{bj}}{2} \left(\frac{1}{r - i\omega_{bj}} + \frac{1}{r + i\omega_{bj}} \right)$$

$$G_{Ea}^S(Q^2) = \frac{4\pi}{2iQ} \int_0^\infty dr\, (e^{iQr} - e^{-iQr})\, r\, \omega_a(r)$$

$$= \frac{\pi\, \omega_{a0}}{iQ} \sum_{j=1}^{3} \Omega_{aj} \int_0^\infty dr \left[\frac{e^{-(3m_\pi - iQ)r}}{r - i\omega_{aj}} - \frac{e^{-(3m_\pi + iQ)r}}{r - i\omega_{aj}} + \frac{e^{-(3m_\pi - iQ)r}}{r + i\omega_{aj}} - \frac{e^{-(3m_\pi + iQ)r}}{r + i\omega_{aj}} \right].$$

$$\int_0^\infty dr\,\frac{e^{-zr}}{r+c} = e^{cz}\,E_1(cz) \equiv G_0(cz)$$

$$E_1(z) = -\gamma - \ln z - \sum_{n=1}^{\infty} \frac{(-1)^n\, z^n}{n\,n!}$$

$$G_{Ea}^{S}(Q^2) = \frac{\pi\,\omega_{a0}}{i\,Q} \sum_{j=1}^{3} \Omega_{aj}\,\{G_0[-i\,\omega_{aj}\,(3m_\pi - i\,Q)] - G_0[-i\,\omega_{aj}\,(3m_\pi + i\,Q)]$$
$$+ G_0[i\,\omega_{aj}\,(3m_\pi - i\,Q)] - G_0[i\,\omega_{aj}\,(3m_\pi + i\,Q)]\}.$$

$$G_{Eb}^{S}(Q^2) = \frac{\pi\,\omega_{b0}}{Q} \sum_{j=1}^{3} \omega_{bj}\,\Omega_{bj}\,\{G_0[-i\,\omega_{bj}\,(3m_\pi - i\,Q)] - G_0[-i\,\omega_{bj}\,(3m_\pi + i\,Q)]$$
$$- G_0[i\,\omega_{bj}\,(3m_\pi - i\,Q)] + G_0[i\,\omega_{bj}\,(3m_\pi + i\,Q)]\}.$$

Main results of the analytic continuation

- At $Q^2 = -9\, m_\pi^2$ a log singularity develops
- For $Q^2 < -9\, m_\pi^2$ we obtain an imaginary part
- The log singularity is well inside the non-accessible region $-4\, M^2 < Q^2 < 0$
- BIG problem: the behaviour for $Q^2 << -9\, m_\pi^2$ is strongly unstable: a tiny variation of the parameters used to fit the current corresponds to a huge variation of the TLFFs

A possible way-out…

$$\omega_a(r) = \frac{\omega_{a0}\, e^{-3m_\pi r}}{[(r+S)^2+\omega_{a1}^2][(r+S)^2+\omega_{a2}^2][(r+S)^2+\omega_{a3}^2]}$$

$$\omega_b(r) = \frac{\omega_{b0}\, r\, e^{-3m_\pi r}}{[(r+S)^2+\omega_{b1}^2][(r+S)^2+\omega_{b2}^2][(r+S)^2+\omega_{b3}^2]}$$

- the value of S cannot be fixed by parametrizing the currents
- the effect of S is to provide a spectral function which vanishes much faster at $Q^2 \rightarrow$ - infinity
- the numerical instability is strongly suppressed
- the “best” value for S is of order 0.3 fm
- possible interpretation: it provides an effective way to suppress the direct production of on-shell mesons at large and negative Q^2 , it indicates the limit of validity of the Skyrme model

“Conclusions”

- It is possible, at least in principle, to evaluate TLFFs by making an analytic continuation of the SLFFs, by using a totally hadronized model as the Skyrme model
- We do not find any clear reason within the model why at threshold electric and magnetic FFs should be equal
- The strong numerical instabilities in the TL region can be reduced by using a parametrization of the current which provides a suppression of the spectral function at large and negative momenta
- Boosting is still a big problem…