Radiative corrections to EM processes

V.S Fadin

Budker Institute of Nuclear Physics
Novosibirsk

Scattering and annihilation electromagnetic processes;
Trento, Italy, 18 - 22 February, 2013

Contents

- Introduction
- General approaches
  - Soft photon approximation
  - Quasi-real particles
  - Structure function method
- Radiative corrections to $e^{\pm} p$ scattering
- Critical review of known results
- Summary

Soft photon approximation

Soft photon approximation is the most well known and most widely used method of calculation of radiative corrections. Its development was related with the problem of infrared divergencies. To understood structure of corrections containing such divergencies the method of their sunnation in all orders in α was developed
D.R Yennie, S.C Frautschi, H Suura, 1961
But nature of these divergences is clear from classical point of view. According to classical electrodynamics any charged particle radiates under acceleration with finite spectral density of the radiation at $\omega \to 0$. It means that average number

$$d\bar{n}(k) = \frac{dI(k)}{\omega}$$

Evidently it should be valid for $\omega \to 0$.

Soft photon approximation

$$dI(k) = \frac{\alpha}{4\pi^2} d^3k \left| \int_{-\infty}^{+\infty} dt e^{i(\omega t - \vec{k}\vec{r}(t))} \left[\vec{n}\,\vec{v}(t)\right] \right|^2$$

$$v^0(t) = 1; \vec{v}(t) = \theta(-t)\vec{v} + \theta(t)\vec{v}'; \vec{r}(t) = \theta(-t)\vec{v}t + \theta(t)\vec{v}'t$$

$$d\bar{n}(k) = \frac{\alpha}{4\pi^2} \frac{d^3k}{\omega} \sum_{\lambda} \left| [\vec{e}^{(\lambda)*} \times \vec{j}(k)] \right|^2$$

$$\sum_{\lambda} e_i^{(\lambda)*} e_j^{(\lambda)} = \delta_{ij} - \frac{k_i k_j}{\vec{k}^{\,2}}$$

$$j^{\mu}(k) = ie \left(\frac{p_f}{(p_f k)} - \frac{p_i}{(p_i k)} \right)^{\mu}$$

Soft photon approximation

$$d\bar{n}(k) = -\frac{\alpha}{4\pi^2}\frac{d^3k}{\omega}j^\mu(k)j_\mu(k)$$

$$j^\mu(k) = ie\left(\sum_f \frac{Q_f p_f}{(p_f k)} - \sum_i \frac{Q_i p_i}{(p_i k)}\right)^\mu$$

$$\bar{n}(\omega_0) = \int_\lambda^{\omega_0} \frac{d\bar{n}(k)}{d\omega} d\omega$$

$$w(n) = \frac{\bar{n}^n}{n!} e^{-\bar{n}}$$

Soft photon approximation

$$d\sigma^{(n)} = d\sigma^{(0)}_{hard} e^{-\bar{n}(\omega_0)} \prod_{i=1}^{n} d\bar{n}(k_i)$$

For emission of any number of soft photons

$$\sum_{n=0}^{\infty} d\sigma^{(n)} = d\sigma_{hard} e^{-\bar{n}(\omega_0)} \frac{\bar{n}(\omega_0)}{n!} = d\sigma_{hard}$$

At experimental restriction

$$\omega < \Delta\epsilon < \omega_0$$

ω_0 – the limit of applicability of the soft photon approximation

$$d\sigma_{\exp}(\Delta\epsilon) = d\sigma^{(0)}_{Born} \exp\left[-\left(\bar{n}(\omega_0) - \bar{n}(\Delta\epsilon)\right)\right]$$

$$= d\sigma^{(0)}_{Born} \exp\left[-\int_{\Delta\epsilon}^{\omega_0} \frac{d\bar{n}(k)}{d\omega} d\omega\right]$$

Soft photon approximation

The last formula can be modified in order to take into account loss of energy by radiated particle. It is useful in the case when "hard" cross section has sharp dependence on energy, for example, for in a resonance region, where

$$\sigma^R_{hafd}(s) = \frac{12\pi\Gamma_{R\to e^+e^-}\Gamma_R}{(s-M_R^2)^2+M_R^2\Gamma_R^2},$$

At $\sqrt{s} \simeq M_R$ essential scale is Γ_R. Emission leads to $s \to (p_- + p_+ - \sum k_i)^2 \simeq s(1-x)$, $x = \Delta\epsilon/\epsilon$, $\Delta\epsilon = \sum \omega_i$. Usual formula valid at $(\Delta\epsilon)^2 \ll \Gamma_R^2 + (2\epsilon - M_R)^2$. Outside passing to x

$$d\sigma(s) = \int_0^1 dx\,\sigma_{hard}(s(1-x))\frac{d\bar{n}(x)}{dx}e^{-(\bar{n}(1)-\bar{n}(x))}$$

$$\bar{n}(1) - \bar{n}(x) = \ln(1/x)\int \frac{\omega^3 d\bar{n}(k)}{d^3k}d\Omega\,.$$

Soft photon approximation

For $e^+e^- \to \gamma^* \to f$ near resonance

$$\sigma(s) = \beta \int_0^1 \frac{dx}{x} x^\beta \sigma_{hard}(s(1-x))$$

$$\beta = \frac{\alpha}{\pi}\left(\ln\left(\frac{s}{m^2}\right) - 1\right).$$

What is seen from this exercise? That soft, truly soft photon approximation is completely determined by the classical value $d\bar{n}(k)$.
Let as see how it looks in QED.

Soft photon approximation

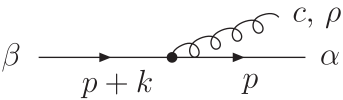
Soft photon emission does not depend on particle structure, in particular on its spin. Charge emission is defined by

$$-\frac{e}{m}(\vec{p}\vec{A})$$

Spin (magnetic moment) emission

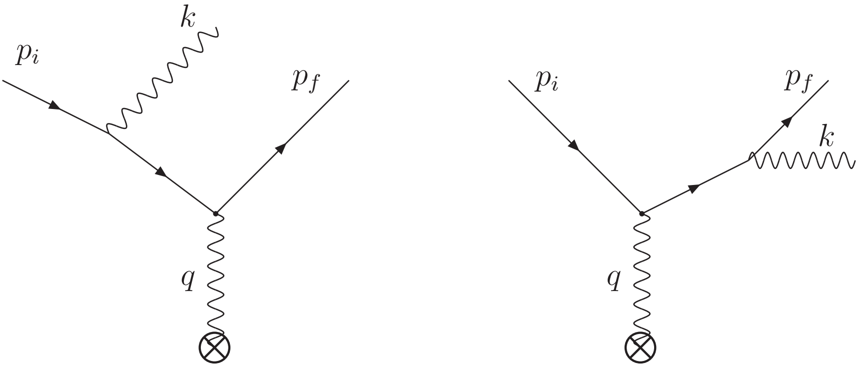
$$-(\vec{\mu}\vec{H}) = -(\vec{\mu}[\vec{\nabla} \times \vec{A}]) = i(\vec{k}[\vec{\mu} \times \vec{A}])$$

$$= \quad i\frac{\delta^{\alpha\beta}}{2p\sum k_i + i0} \,,$$



$$= \quad -ie\delta^{\alpha\beta}2p^{\rho} \,.$$

For one charge particle



$$M^{(1)} = (-ie)e^*_\mu(k)j^\mu(k)M^{(0)},$$

Reason of the factorization: singular in k part of matrix elements come from space-time infinity.

Soft photon approximation

$$j^\mu(k) = ie\left(\frac{p_f}{(p_f k)} - \frac{p_i}{(p_i k)}\right)^\mu$$

$$d\sigma^{(1)} = d\sigma^{(0)} dW(k)$$

$$d\sigma^{(n)} = d\sigma^{(0)} \prod_{i=1}^{n} dW(k_i)$$

$$dW(k) = \frac{\alpha}{4\pi^2}\frac{d^3k}{\omega}\sum_\lambda \left|e_\mu^{(\lambda)*} j^\mu(k)\right|^2$$

$W(k)$ is usually called emission probability. In fact, it is not so. It is clear, that $dW(k) = d\bar{n}(k)$, that it is mean photon number, and emission probability differs from $W(k)$ by the factor

$$e^{-\int dW(k)\theta(\omega_0 - \omega)}$$

Soft photon approximation

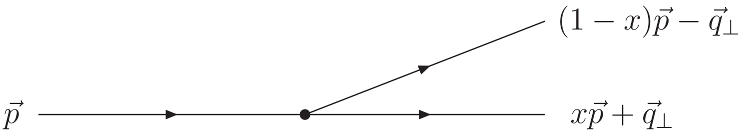
We found this factor from classical consideration, from the requirement of the Poisson distribution. In QED it this factor comes from virtual corrections. Using the simplified diagram technique, one obtains

$$M^{(0)} = M_B^{(0)}\, e^{-\int \frac{d^4k}{(2\pi)^4 i}\frac{1}{k^2+i0} j_\mu(k) j^\mu(k)}$$

with the prescription that only the contribution of the pole at $k^2 = 0$ must be taken into account.
Instead, one can modify the the integrand in such a way that it becomes convergent. But in this way one inevitably oversteps the limits of strict soft photon approximation.
This going out the limits can be done in different ways. Some of them can seem reasonable and others not reasonable. The standard way is retaining $k^2 + i0$ beside (kp) in denominators of $\jmath_\mu(k)$.

Quasi-real particles

The structure function method is based on collinear factorization.
The reason of factorization — difference in essential distances.
For collinear emission:



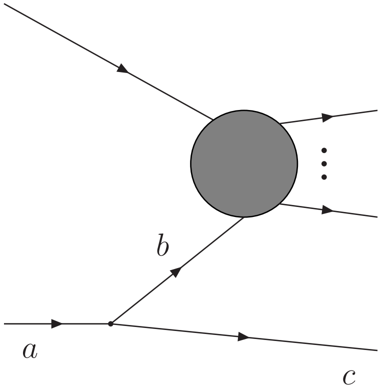
$$l_{eff} \sim \tau_{eff} \sim (\Delta\epsilon)^{-1}$$

$$\Delta\epsilon = \sqrt{(1-x)^2\vec{p}^{\,2}+\vec{q}^{\,2}+m_2^2}+\sqrt{x^2\vec{p}^{\,2}+\vec{q}^{\,2}+m_1^2}-\sqrt{\vec{p}^{\,2}+m^2}$$
$$\simeq \frac{\vec{q}_\perp^{\,2}+m_2^2x+m_1^2(1-x)-m^2x(1-x)}{2x\,(1-x)\,p}$$

Quasi-real particles

If in the "soft" stage of interaction is the decay $a \to b + c$, and c is one of final particles, while b participated in "hard" stage, then

$$S_{fi} = \sum_b \frac{M_a^{bc}}{\Delta\epsilon} \frac{1}{2\epsilon_b} S_b$$



Quasi-real particles

$$d\sigma_a(\vec{p}) = dn_a^b(x, \vec{q}_\perp) d\sigma_b(x\vec{p}) \ ,$$

$$dn_a^b(x, \vec{q}_\perp) = \frac{1}{2\epsilon_a} \overline{\sum} \frac{\left|M_a^{bc}\right|^2}{(\Delta\epsilon)^2} \frac{d^3 p_b}{2\epsilon_b (2\pi)^3} \frac{d^3 p_c}{2\epsilon_c (2\pi)^3} (2\pi)^3 \delta(\vec{p}_A - \vec{p}_B - \vec{p}_C) \ ;$$

Generally speaking, the particle b is polarized with the density matrix

$$\rho_{bb'} = \left(\overline{\sum_{a,c}} M_a^{bc} (M_a^{b'c})^* \right) / \left(\overline{\sum_{a,b,c}} \left| M_a^{bc} \right|^2 \right) \ ;$$

Quasi-real particles

$$dn_e^\gamma(x,\vec{q}_\perp) = \frac{\alpha}{2\pi}\frac{dx}{x}\frac{d^2q_\perp}{\pi}\frac{\left[\vec{q}_\perp^{\,2}(1+(1-x)^2)+m^2x^4\right]}{\left(\vec{q}_\perp^{\,2}+m^2x^2\right)^2}$$

$$dn_\gamma^e(x,\vec{q}_\perp) = \frac{\alpha}{2\pi}dx\frac{d^2q_\perp}{\pi}\frac{\left[\vec{q}_\perp^{\,2}(1+(1-x)^2)+m^2\right]}{\left(\vec{q}_\perp^{\,2}+m^2\right)^2}\,.$$

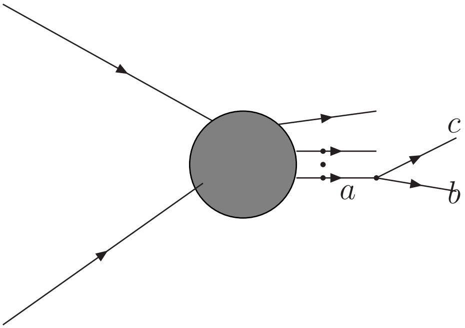
Note that in the region of small $\vec{q}^{\,2}$ accuracy of above formulas can be rather high: rejected terms are of order $(m^2+\vec{q}^{\,2})/Q^2$, Q is the hard scale.

But when the particle c is not detected we have to integrate over $\vec{q}$. At $\vec{q}^{\,2} \sim Q^2$ the approximation becomes invalid, so that one put Q^2 as the up limit of the integration. The accuracy becomes only logarithmic.

Quasi-real particles

If ”soft” stage is the decay $a \to b + c$ of the particle a created in ”hard” stage, then

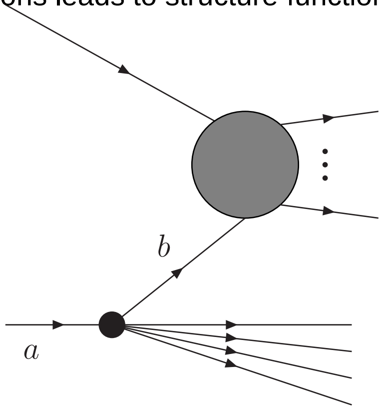
$$S_{fi} = -\sum_a S_a \frac{1}{2\epsilon_a} \frac{M_a^{bc}}{\Delta\epsilon}$$



$$d\sigma_b = d\sigma_a(\vec{p}) dn_a^b(x, \vec{q}_\perp)$$

Structure functions

Iteration of integrated quasi-real particle formulas with account of virtual corrections leads to structure functions.



$$d\sigma_a(s) = \sum_b \int_0^1 dx f_a^b(x, Q^2) \hat{\sigma}_b(xs)$$

Structure functions

The structure function $f_a^b(x, Q^2)$ take into account corrections containing large logarithms of the type $\ln Q^2/m^2$. It obeys the equation

$$\frac{df_a^b(x,Q^2)}{d\ln Q^2} = \frac{\alpha}{2\pi}\int_x^1 \frac{dz}{z} P_a^c\left(\frac{x}{z}\right) f_c^b(z,Q^2)$$

where

$$\frac{\alpha}{2\pi}P_a^b(x) = \frac{d}{d\ln Q^2}\int_0^{Q^2} \frac{dn(x,\vec{q}^{\,2})}{d\vec{q}^{\,2}} d\vec{q}^{\,2}$$

One can be disappointed by this picture because of habit to Feynman gauge where large logarithmic corrections come from fhoton exchange between particles with srtongly different momenta. But, as it often occurs, the Feynman gauge obscures the physical picture. In physical gauges leading corrections come from diagrams with photon vertices on the line of same particles.

Structure functions

Applying the same procedure to second colliding particle we obtain

$$d\sigma_{AB}(s) \;=\; \sum_{a,b}\int_0^1 dx_a \int_0^1 dx_b f_A^a(x_a,Q^2) f_B^b(x_b,Q^2)\hat{\sigma}_{ab}(x_a x_b s)$$

Analogous procedure cann be applied to final state

$$\frac{d\sigma^b}{dx_b} = \sum_a \int_x^1 \frac{dz}{z}\bar{f}_a^b(\frac{x}{z},{}^2)\frac{d\sigma_a}{dz}\;,$$

$$\frac{d\bar{f}_a^b(x,Q^2)}{d\ln q^2} = \frac{\alpha_s(q^2)}{2\pi}\int_x^1 \frac{dz}{z}\bar{P}_a^c(\frac{x}{z})\bar{f}_c^b(z,q^2)$$

In leading logarithmic approximation they are simply coincide

$$\bar{P}_b^a(z) = \bar{P}_b^a(z)$$

Structure functions

Evident generalizations:

$$d\sigma_{AB}(s) = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b f_A^a(x_a, Q^2) f_B^b(x_b, Q^2) \hat{\sigma}_{ab}(x_a x_b s, Q^2)$$

$$\frac{d\sigma_{AB}^C(s)}{dx_C} = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \int_x^1 \frac{dz}{z}$$

$$f_A^a(x_a, Q^2) f_B^b(x_b, Q^2) \bar{f}_c^C(\frac{x}{z}, Q^2) \frac{d\hat{\sigma}_{ab}^c(x_a x_b s)}{dz}$$

I think that, in principle, the structure function method for calculation of EM corrections is convenient and powerful. But, by definition, it does not take into account interference effects, such as two-phonon exchange in *ep* scattering.

ep scattering

There is an evident discrepancy between results of measurements of G_E/G_M by two methods:
Rosenbluth separation

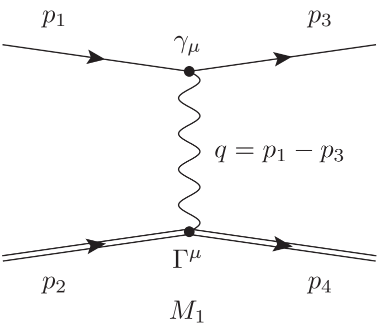
$$\frac{G_E^2}{G_M^2} = \tau\frac{f'(\epsilon)}{f(0)}, \quad \tau = \frac{Q^2}{4M^2}, \quad f(\epsilon) = \frac{1}{\tau+\epsilon}\left(\frac{d\sigma}{d\Omega}\right)^{-1}_{point}\left(\frac{d\sigma^B}{d\Omega}\right),$$

$\epsilon = \left(1 + 2(1+\tau)\tan^2(\theta/2)\right)^{-1}$ – the virtual photon polarization parameter, and polarization transfer at scattering of polarized electron beam on unpolarized target

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\epsilon)}{2\epsilon}}\frac{P_T}{P_L},$$

$P_T(P_L)$ — the polarization of the recoil proton transverse (longitudinal) to the proton momentum in the scattering plane. Formulas above are obtained in the Born approximation.

ep scattering



$$\Gamma^\mu(q) = F_1(Q^2)\,\gamma^\mu + F_2(Q^2)\,\frac{i\sigma^{\mu\nu}q_\nu}{2M},$$

$Q^2 = -q^2,\;\; G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2),\;\; G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$

The cause of the discrepancy must be understood.
One possible cause is the radiative corrections.
Most dangerous they are for the Rosenbluth method.
At large Q^2 contribution of the term with G_E^2 to the cross section becomes small.
It makes determination of G_E very sensitive to ϵ-dependent corrections.

Account of the radiative corrections in

[1] R. C. Walker et al., Phys. Rev. D 49, 5671 (1994)
[2] J. Arrington, Phys. Rev. C 68, 034325 (2003)
[3] M. E. Christy et al., Phys. Rev. C 70, 015206 (2004)
[4] I. A. Qattan et al., Phys. Rev. Lett. 94, 142301 (2005)

must be analyzed.

ep scattering

In these papers, the main theoretical source of the radiative corrections is

Y. -S. Tsai, Phys. Rev. **122**, 1898 (1961)

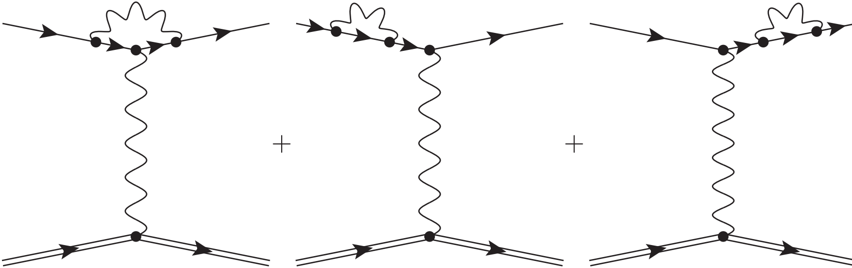
L. W. Mo and Y. -S. Tsai, Rev. Mod. Phys. **41**, 205 (1969).

In the following we will call it MoT.
The radiative corrections in this source have factored form:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^B}{d\Omega}(1+\delta)$$

To distinguish corrections of different type let us denote electron charge e and proton one $-Ze$.
There are virtual (corresponding to elastic process) and real (accounting inelasticity) corrections.

The first order virtual correction proportional to Z^0 come from interference of the Born diagram with the diagrams



M_e

Virtual electron correction

$$\delta^{ve}_{MoT} = \frac{Z^0\alpha}{\pi}\left(-K(p_1,p_3)+K(p_1,p_1)+\frac{3}{2}\ln\left(\frac{-q^2}{m^2}\right)-2\right),$$

where

$$K(p_i,p_j) = \frac{2(p_i\cdot p_j)}{-i\pi^2}\int\frac{d^4k}{(k^2-\lambda^2+i0)(k^2-2(k\cdot p_i)+i0)(k^2-2(k\cdot p_j)+i0)}$$

$$= (p_i\cdot p_j)\int_0^1\frac{dy}{p_y^2}\ln\left(\frac{p_y^2}{\lambda^2}\right),$$

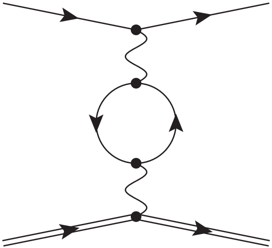
$p_y = y\,p_i + (1-y)\,p_j$. The only approximation in calculation of this correction is smallness of m^2. Explicitly,

$$\delta^{ve}_{MoT} = \frac{Z^0\alpha}{\pi}\left\{-\left(\ln\left[\frac{-q^2}{m^2}\right]-1\right)\ln\left[\frac{m^2}{\lambda^2}\right]-\frac{1}{2}\ln^2\left[\frac{-q^2}{m^2}\right]+\frac{\pi^2}{6}+\right.$$

$$\left.+\frac{3}{2}\ln\left[\frac{-q^2}{m^2}\right]-2\right\}$$

Vacuum polarization correction

The interference of the Born diagram and the diagram with vacuum polarization gives also correction proportional to Z^0



M_v

Vacuum polarization correction

Here, besides electron, muon and τ-lepton loops, hadron vacuum polarization must be accounted.
For light leptons

$$\delta^{ll}_{MoT} = \frac{Z^0\alpha}{\pi}\left(\frac{2}{3}\ln\left[\frac{-q^2}{m^2}\right] - \frac{10}{9}\right),$$

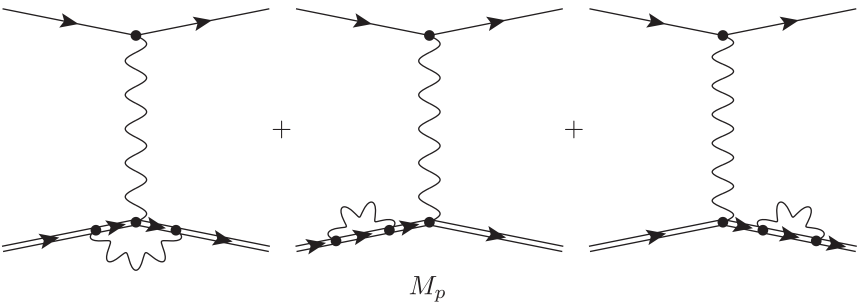
for heavy

$$\delta^{hl}_{MoT} = \frac{2Z^0\alpha}{\pi}\left[\frac{1}{9} - \frac{2q^2+4m^2}{3t}\left(1 - \sqrt{\frac{4m^2-q^2}{-q^2}}\ln\left(\sqrt{-\frac{q^2}{4m^2}} + \sqrt{1-\frac{q^2}{4m^2}}\right.\right.\right.$$

These expressions also are well known.
The hadron contribution to the vacuum polarization can not be presented in an analogous form, because it includes strong interaction effect. It is calculated using dispersion relations and $e^+e^- \rightarrow$ *hadrons* experimental data. Now this contribution is well known.

Virtual proton correction

The virtual correction proportional to Z^2 come from interference of the Born diagram with the diagrams



Virtual proton correction

This contribution can not be calculated "from the first principles". They are evaluated by MoT using the soft photon approximation

$$\delta^{vp}_{MoT} = \frac{Z^2\alpha}{\pi}\left(-K(p_2,p_4) + K(p_2,p_2)\right).$$

Other evaluations are possible. In the paper
L. C. Maximon and J. A. Tjon, Phys. Rev. C **62** (2000) 054320
which is called in the following MTj, this correction is calculated using dipole or monopole form factors

$$F_1(Q^2) = F_2(Q^2) = \left(\frac{\Lambda^2}{\Lambda^2 + Q^2}\right)^n, \quad n = 1, 2$$

Therefore

$$\delta^{vp}_{MTj} = \delta^{vp}_{MoT} + \delta^{(1)}_{el},$$

where $\delta^{(1)}_{el}$ is infrared finite.

Virtual proton correction

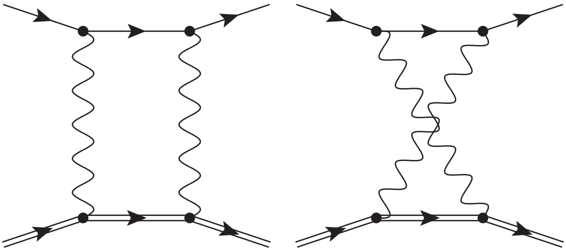
For the dipole parametrization with $\Lambda = 700$ MeV/c

$\delta_{el}^{(1)} = 0.0116$ for $Q^2 = 16$ (GeV/c)^2

In any case, this correction depends only on Q^2.

Virtual electron-proton correction

The most interesting virtual, proportional to Z^1 corrections, come from the interference of the Born diagram with the diagrams



M_{ep}

Virtual electron-proton correction

In their evaluation MoT used the soft photon approximation with additional simplification.

$$\delta^{vep}_{MoT} = \frac{Z^2\alpha}{\pi}\left(-K(p_1,p_2) - K(p_3,p_4 + K(p_1,p_4) + K(p_2,p_3)\right),$$

whereas in the soft photon approximation

$$\delta^{vep}_{sf} = \frac{Z^2\alpha}{\pi} Re\left(-K(p_1,-p_2) - K(p_3,-p_4 + K(p_1,p_4) + K(p_2,p_3)\right).$$

and

$$-Re[K(p_1,-p_2)] - Re[K(p_3,-p_4)] + K(p_1,p_2) + K(p_3,p_4)$$

$$= \pi^2 - 2\int_{1-\frac{M}{2\epsilon_1}}^{1+\frac{M}{2\epsilon_1}} \frac{dx}{x} \ln|1-x|$$

Virtual proton correction

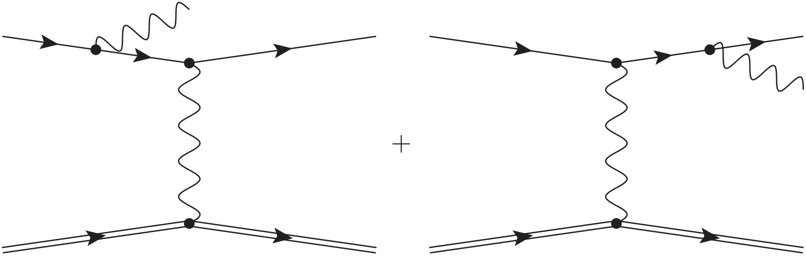
But the soft photon approximation in its standard form is not good here.
More reliable approximation is used by MTj.

$$\delta_{MTj}^{vep} - \delta_{MoT}^{vep} = 2\,Z\,\frac{\alpha}{\pi}\left(-\ln\frac{\epsilon_1}{\epsilon_3}\ln\left[\sin\left[\frac{\theta}{2}\right]^2\right] - Li_2\left[1-\frac{2\epsilon_3}{M}\right]\right.$$

$$\left. + Li_2\left[1-\frac{2\epsilon_1}{M}\right]\right).$$

Real electron correction

The first order real correction proportional to Z^0 come from the diagrams



M_e^r

Real electron correction

$$\delta^{re}_{MoT} = \frac{Z^0\alpha}{\pi}\left(\left(\ln\left[\frac{-q^2}{m^2}\right] - 1\right)\ln\left[\frac{\omega^2 m^2}{\lambda^2\epsilon_1\epsilon_3}\right] + \frac{1}{2}\ln\left[\frac{-q^2}{m^2}\right]^2\right.$$

$$\left. - \frac{\ln[\eta]^2}{2} - \frac{\pi^2}{6}\right),$$

where

$$\eta = 1 + \frac{\epsilon_1}{M}(1 - \cos\theta))$$

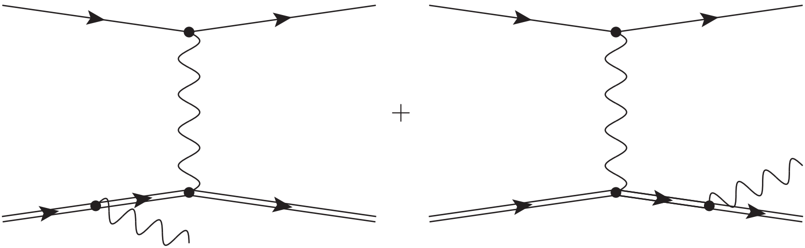
Here the term

$$-\frac{\alpha}{\pi}\left[\frac{\pi^2}{6} - \Phi[cos^2(\theta/2)]\right]$$

was omitted in papers MoT. Later on was restored in
Y. -S. Tsai, SLAC-PUB-0848, (1971)
and included in experimental papers as δ_{Sch} (Schwinger's correction)).

Virtual proton correction

The real correction proportional to Z^2 come from the diagrams



M_p^r

Real proton correction

This contribution also can not be calculated ”from the first principles”. They are evaluated by using the soft photon approximation.

$$\delta^{rp}_{MoT} = Z^2\frac{\alpha}{\pi}\left(\left(\frac{\epsilon_4}{|\mathbf{p}_4|}\ln[x]-1\right)\ln\left[\frac{4\omega^2}{\lambda^2}\right]-\right.$$

$$\left.-\frac{\epsilon_4}{|\mathbf{p}_4|}\left(\ln[x]^2+Li_2\left[-\frac{1}{x^2}\right]+\frac{\pi^2}{12}\right)+\ln\left[\frac{2\epsilon_4}{M}\right]+\ln[2]\right)$$

But in derivation of this result there is the error. Correct result is given by MTj.

$$\delta^{rp}_{MTj}-\delta^{rp}_{MoT} = Z^2\frac{\alpha}{\pi}\left(-\frac{\epsilon_4}{|\mathbf{p}_4|}\left(Li_2\left[1-\frac{1}{x^2}\right]-Li_2\left[-\frac{1}{x^2}\right]-\frac{\pi^2}{12}\right)+\right.$$

$$\left.+\frac{\epsilon_4}{|\mathbf{p}_4|}\ln[x]+1-\ln\left[\frac{2\epsilon_4}{M}\right]-\ln[2]\right)$$

Real electron-proton correction

The most interesting real correction, proportional to Z^1, comes from the interference diagrams with electron and photon emission.

$$\delta^{rep}_{MoT} = 2Z\frac{\alpha}{\pi}\left(\ln[\eta]\ln\left[\frac{4\omega^2}{\lambda^2}\right] + \frac{1}{2}Li_2\left[1-\eta\frac{2\epsilon_4}{M}\right] - \frac{1}{2}Li_2\left[1-\frac{1}{\eta}\frac{2\epsilon_4}{M}\right]\right)$$

Again there is an error here. Correct result is given by MTj

$$\delta^{rep}_{MTj} - \delta^{rep}_{MoT} = 2\,Z\,\frac{\alpha}{\pi}\left(-\ln[\eta]\ln[x] - Li_2\left[1-\frac{1}{\eta\,x}\right] + Li_2\left[1-\frac{\eta}{x}\right] - \right.$$

$$\left. -\frac{1}{2}\,Li_2\left[1-\eta\frac{x^2+1}{x}\right] + \frac{1}{2}\,Li_2\left[1-\frac{x^2+1}{\eta\,x}\right]\right)$$

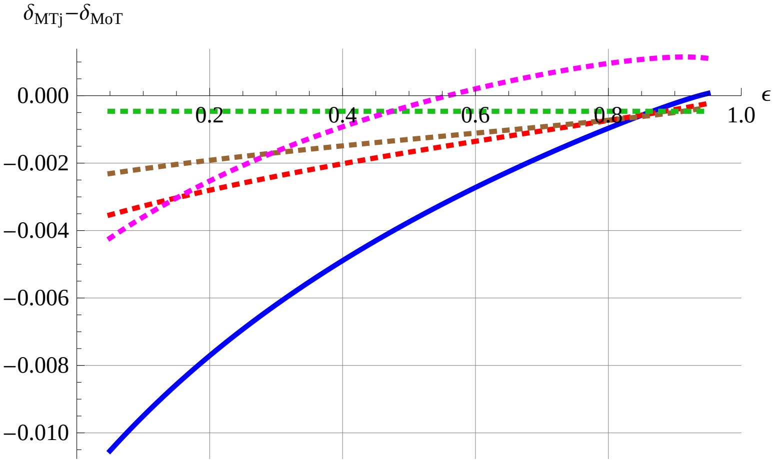


Figure : Difference at $Q^2 = 1\ GeV^2$.

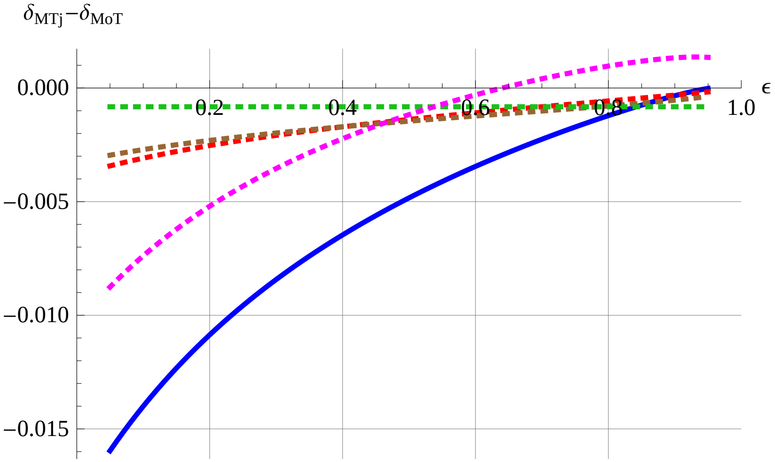


Figure : Difference at $Q^2 = 3\ GeV^2$.

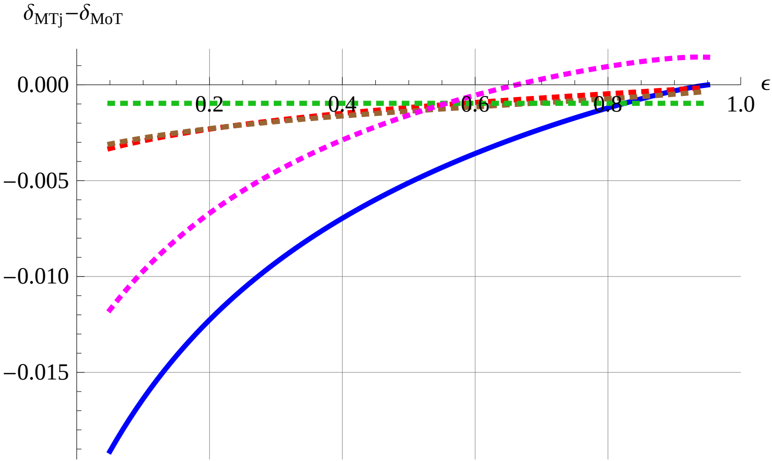


Figure : Difference at $Q^2 = 5\ GeV^2$.

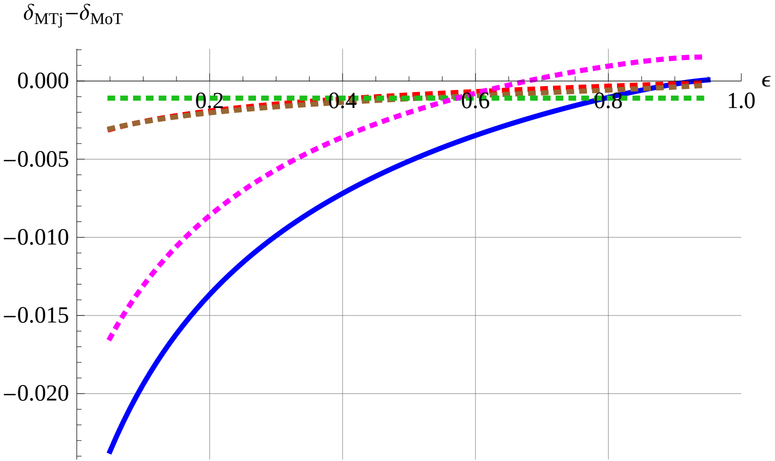


Figure : Difference at $Q^2 = 10\ GeV^2$.

Summary

- The strict soft photon approximation gives purely classical result.
- All attempts to obtain more are model dependent.
- Structure function method is convenient and powerful.
- But it can not help in calculation of two-phonon exchange in *ep* scattering.
- There are evident shortages in account of radiative corrections at extraction of form factors by Rosenbluth methods.
- It is desirable to understand influence of these shortages on the result.
- It is very desirable to perform accurate account of radiative corrections in two-photon exchange experiments in such a way that it will be possible to connect results of different experiments.