Constituent
Quark Models
and electromagnetic form factors

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Trento, February 2013

Outline of the talk

- The Model (hCQM)
- The helicity amplitudes
- The elastic e.m. form factors of the nucleon
- The Unquenched Quark Model (higher Fock components in a systematic way)
- The axial form factor in SL and TL(PANDA & BES? and Why?)

The Model
(hCQM)

hypercentral **C**onstituent **Q**uark **M**odel

Hypercentral Constituent Quark Model
hCQM

free parameters fixed from the spectrum

Comment
The description of the spectrum is the first task of a model builder

Predictions for:
photocouplings
transition form factors
elastic from factors
....

describe data (if possible)
understand what is missing

LQCD (De Rújula, Georgi, Glashow, 1975)

the quark interaction contains

a long range spin-independent confinement

a short range spin dependent term

Spin-independence $\longrightarrow$ SU(6) configurations

SU(6) configurations for three quark states

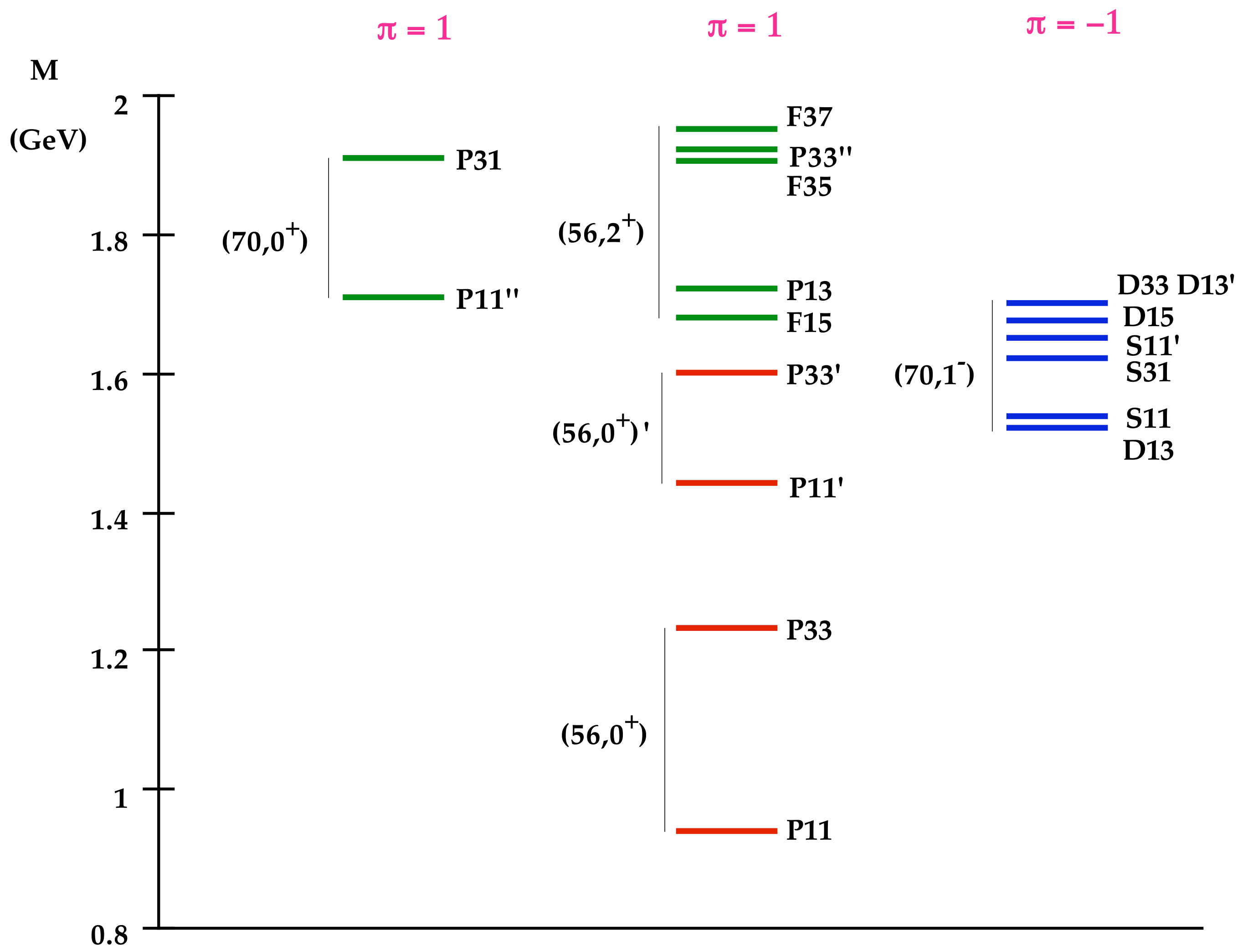
$6 \times 6 \times 6 =$	20 +	70 +	70 +	56
	A	M	M	S

Notation

(d, L^{π})

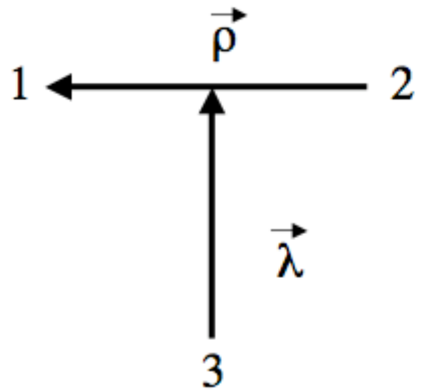
d = dim of SU(6) irrep
L = total orbital angular momentum
π = parity

PDG 4* & 3*



π = 1
π = 1
π = −1
M
(GeV)
2
1.8
1.6
1.4
1.2
1
0.8
P31
(70,0+)
P11''
F37
P33''
F35
(56,2+)
P13
F15
P33'
(56,0+)'
P11'
P33
(56,0+)
P11
D33 D13'
D15
S11'
S31
(70,1-)
S11
D13

Jacobi coordinates



Hyperspherical Coordinates

$$(\rho, \Omega_\rho, \lambda, \Omega_\lambda) \Rightarrow (x, t, \Omega_\rho, \Omega_\lambda)$$

$$x = \sqrt{\rho^2 + \lambda^2}$$ hyperradius

$$t = arctg \frac{\rho}{\lambda}$$ hyperangle

$\gamma = 2n + l_\rho + l_\lambda$

$$L^2(\Omega) Y_{[\gamma]}(\Omega) = -\gamma(\gamma + 4) Y_{[\gamma]}(\Omega)$$

$$L^2(\Omega) \Leftrightarrow C_2(O(6))$$

γ grand angular quantum number

$Y_{[\gamma]}(\Omega)$ Hyperspherical harmonics

$$\sum_{i<j} V(\mathbf{r}_{ij}) \approx V(x) + \dots$$

$\gamma = 2n + l_\rho + l_\lambda$

Hasenfratz et al. 1980:
$\Sigma\, V(r_i, r_j)$ is approximately hypercentral

Hypercentral Hypothesis $V = V(x)$

Factorization

$$\psi(x, t, \Omega_\rho, \Omega_\lambda) = \psi_{\nu\gamma}(x) \quad Y_{[\gamma, l_\rho, l_\lambda]}$$

("dynamics") ("geometry")

Only one differential equation in x (hyperradial equation)

Hypercentral Model

Phys. Lett. B, 1995

$V(x) = -\tau/x + \alpha\, x$

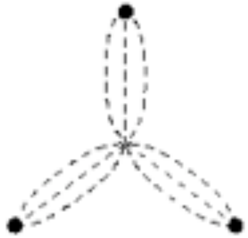
Hypercentral approximation of $V = - b/r + c\, r$

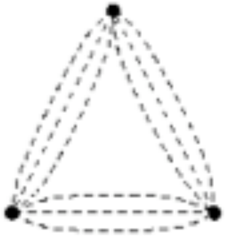
- QCD fundamental mechanism

3-body forces

Carlson et al, 1983
Capstick-Isgur 1986
hCQM 1995

- Flux tube model



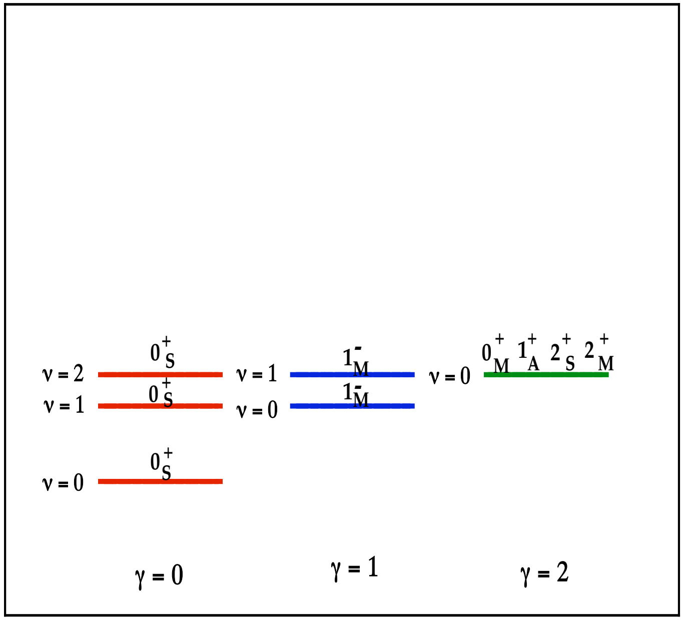


Two analytical solutions

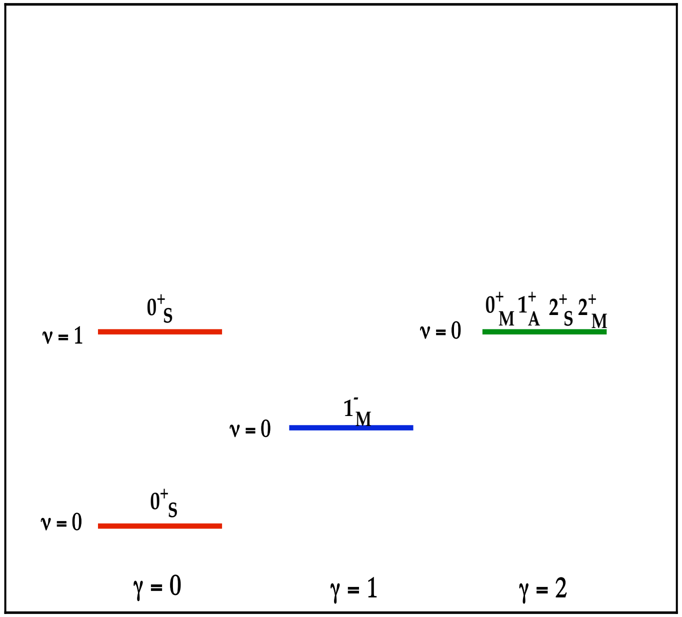
hyperCoulomb $\qquad - \tau/x$

h. o. $\qquad \Sigma_{i<j} 1/2\ k\ (r_i - r_j)^2 = 3/2\ \ k\ x^2$

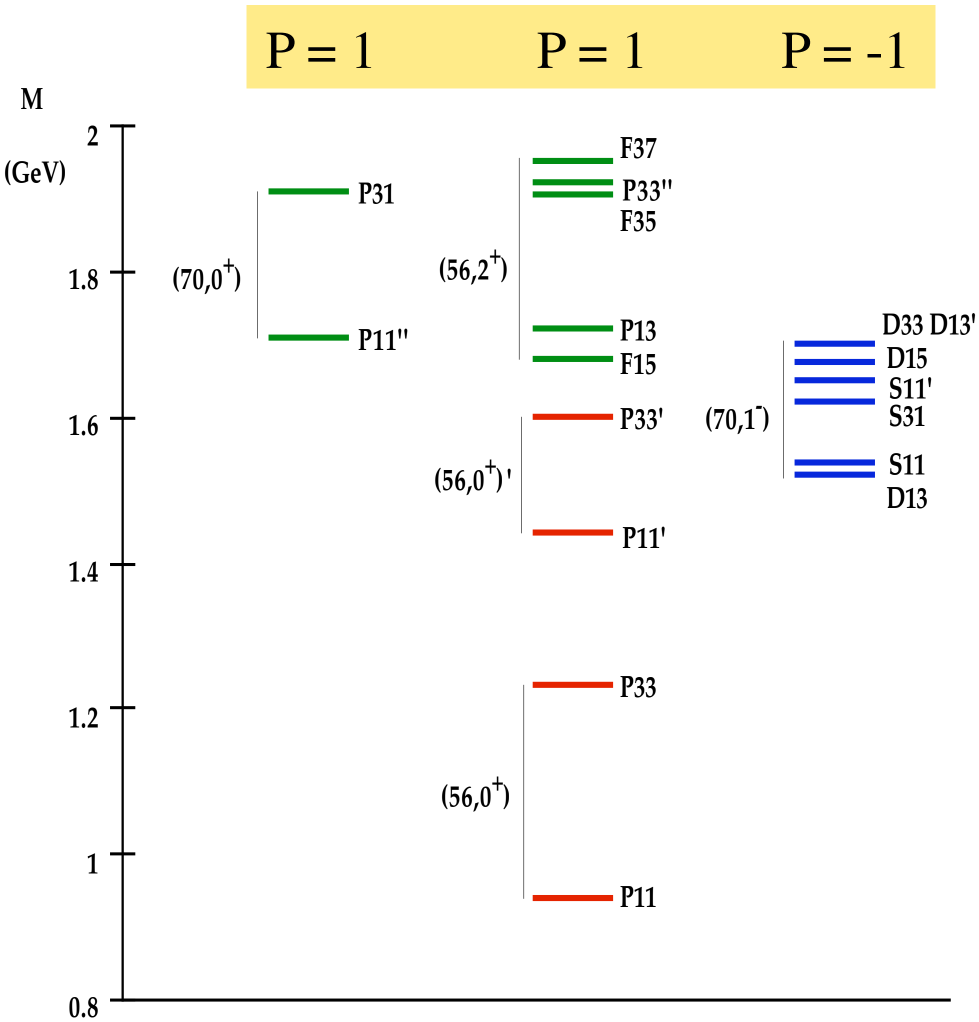
a) HYPERCOULOMB



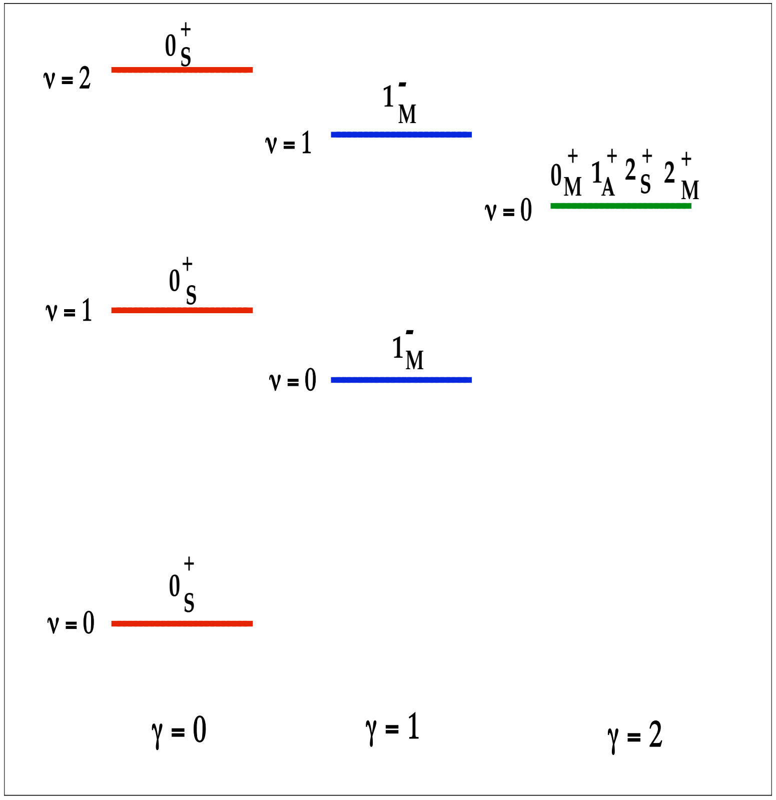
b) H. O.



PDG 4* & 3*

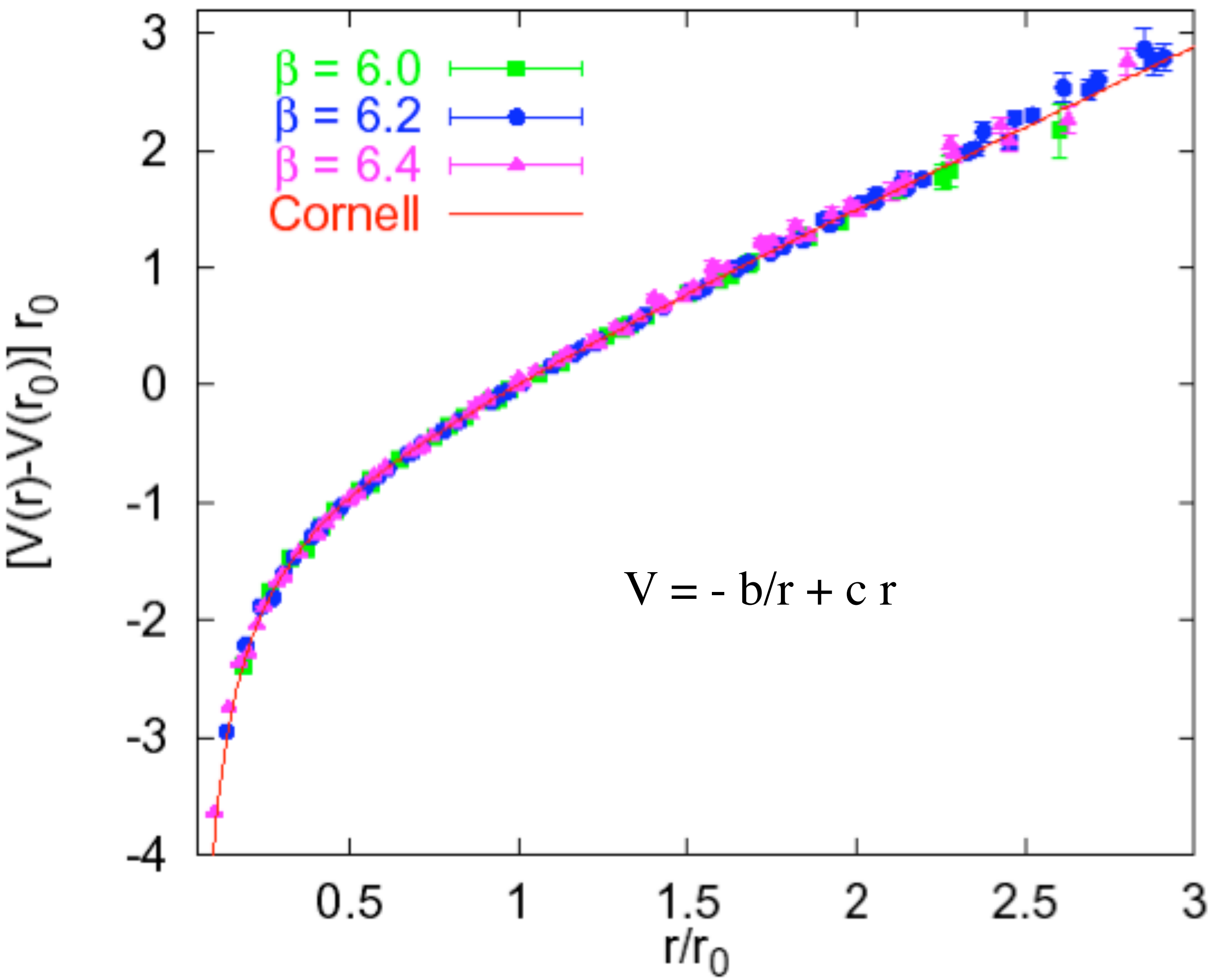


$$V(x) = -\tau/x + \alpha x$$



Quark-antiquark lattice potential

G.S. Bali Phys. Rep. 343, 1 (2001)

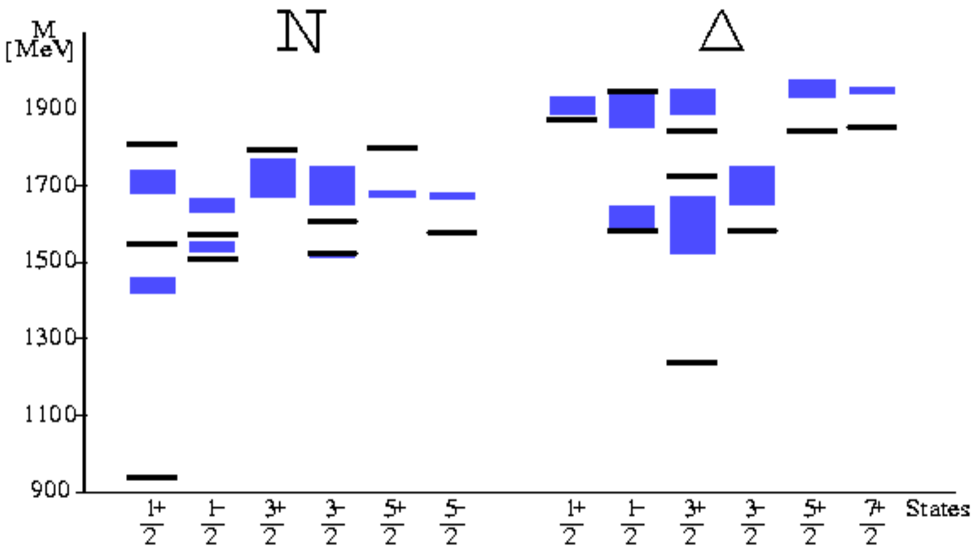


Hypercentral Model (1)

$$H_{3q} = 3m + \sum_{i=1}^{3} \frac{\mathbf{p}_i^2}{2m} + V(x) + H_{hyp}$$

M. Ferraris, M. M. Giannini, M. Pizzo, E. Santopinto, L. Tiator, Phys. Lett. B364 (1995), 231

- $V(x) = -\frac{\tau}{x} + \alpha x;\qquad H_{hyp} = A\left[\sum_{i<j} V^S(\mathbf{r}_i, \mathbf{r}_j)\ \ \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \text{tensor}\right]$
- 3 parameters $\tau\ \alpha\ A$ ← fixed to the spectrum, $m = \frac{M}{3}$



$\tau = 4.59$

$\alpha = 1.61\ \text{fm}^{-1}$

$A \Leftarrow (N - \Delta)$

$$x = \sqrt{\rho^2 + \lambda^2}$$

hyperradius

Results (predictions) with the Hypercentral Constituent Quark Model

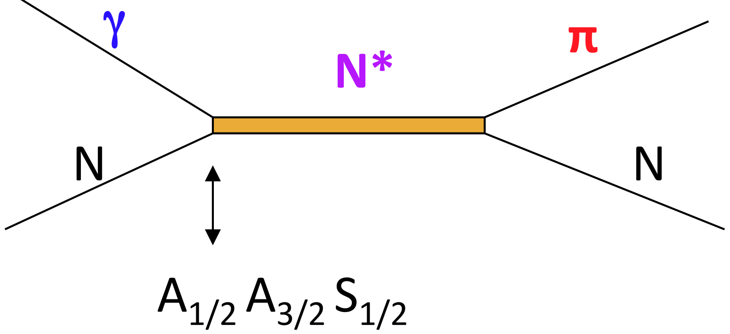
for

- ❑ Helicity amplitudes
- ❑ Elastic nucleon form factors

The helicity amplitudes

HELICITY AMPLITUDES

Extracted from electroproduction of mesons



Definition

$$A_{1/2} = < N^* \; J_z = 1/2 \mid H^T_{em} \mid N \; J_z = -1/2 >$$ §

$$A_{3/2} = < N^* \; J_z = 3/2 \mid H^T_{em} \mid N \; J_z = 1/2 >$$ §

$$S_{1/2} = < N^* \; J_z = 1/2 \mid H^L_{em} \mid N \; J_z = 1/2 >$$

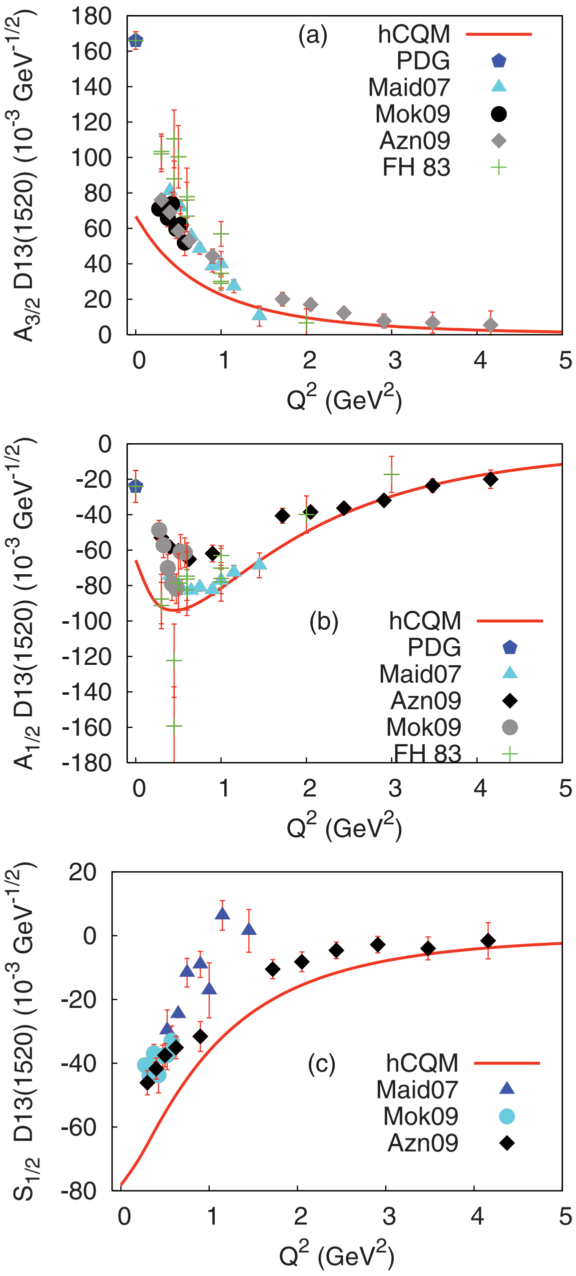
N, N^* nucleon and resonance as 3q states

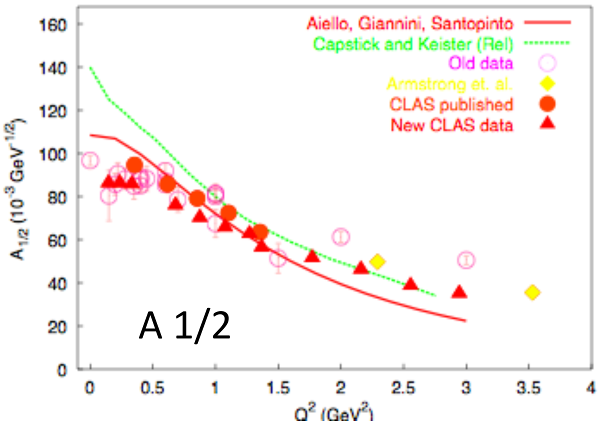
H^T_{em} H^l_{em} model transition operator

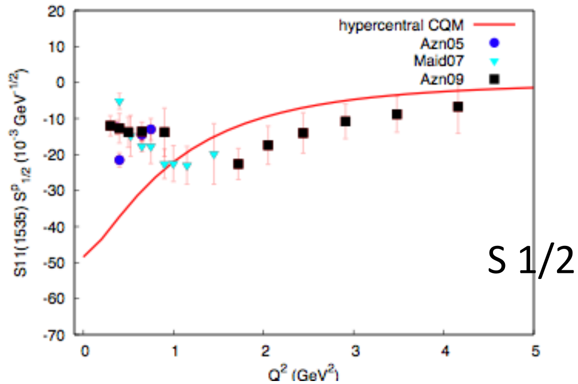
§ results for the negative parity resonances:
M. Aiello, M.Giannini, E. Santopinto J. Phys. G24, 753 (1998)

Systematic predictions for transverse and longitudinal amplitudes
E. Santopinto, M.Giannini, PR C 86, 065202 **(2012)**

D13 transition amplitudes

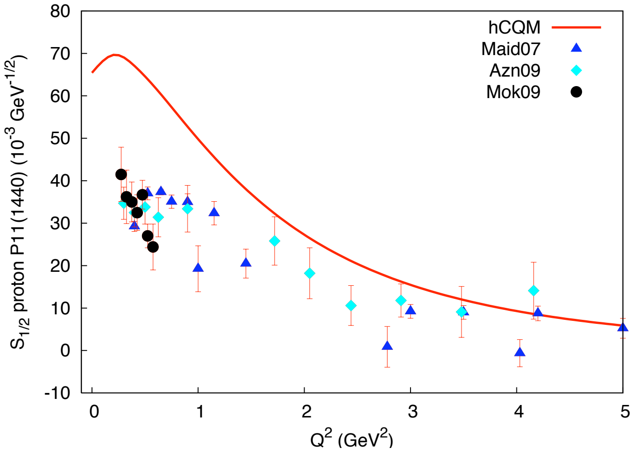




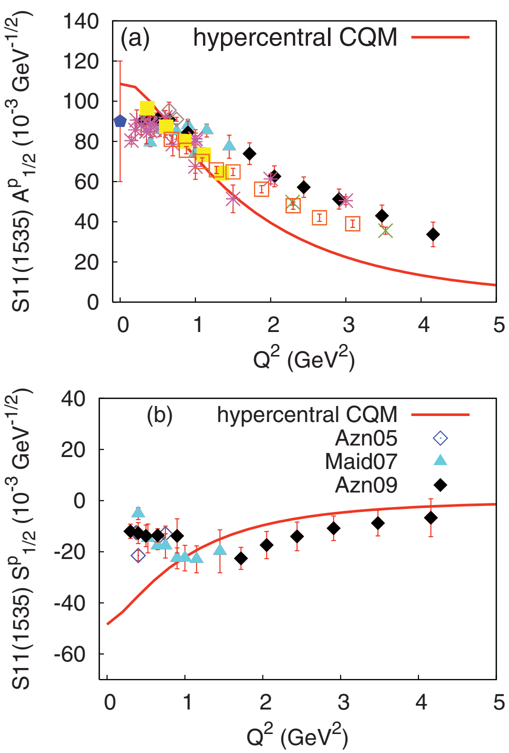


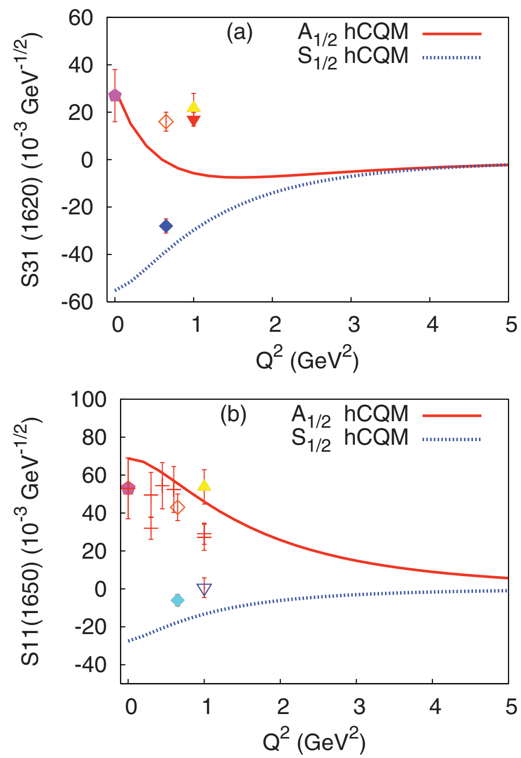
S11(1535) transition amplitudes

Roper N(1440)

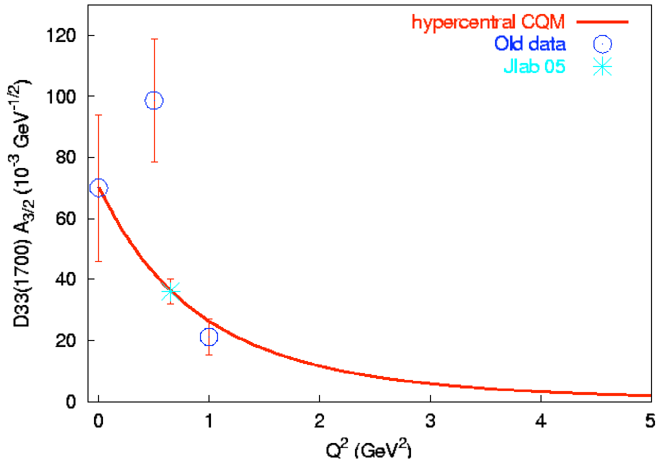


hCQM
Maid07
Azn09
Mok09
S1/2 proton P11(1440) (10-3 GeV-1/2)
Q2 (GeV2)
80
70
60
50
40
30
20
10
0
-10
0
1
2
3
4
5


(a)
hypercentral CQM
S11(1535) $A^p_{1/2}$ (10^{-3} GeV$^{-1/2}$)
Q^2 (GeV^2)
(b)
hypercentral CQM
Azn05
Maid07
Azn09
S11(1535) $S^p_{1/2}$ (10^{-3} GeV$^{-1/2}$)
Q^2 (GeV^2)

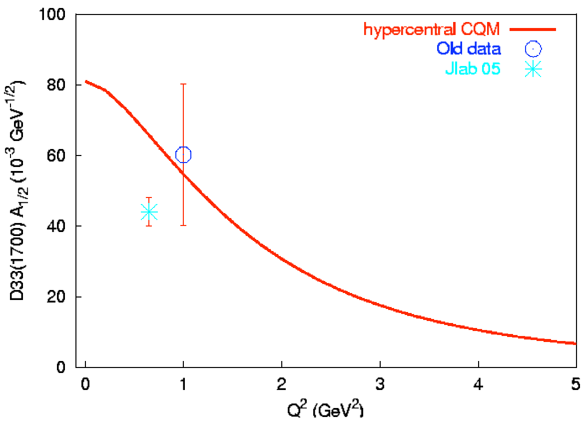

(a)
$A_{1/2}$ hCQM
$S_{1/2}$ hCQM
S31 (1620) (10^{-3} GeV$^{-1/2}$)
Q^2 (GeV^2)
(b)
$A_{1/2}$ hCQM
$S_{1/2}$ hCQM
S11(1650) (10^{-3} GeV$^{-1/2}$)
Q^2 (GeV^2)

D33(1700)



A 3/2

A 1/2



- The hCQM seems to provide realistic three-quark wave functions

- The main reason is the presence of the **hypercoulomb** term

Solvable model

$V(x) = -\tau/x + \alpha\, x$ linear term treated as a perturbation

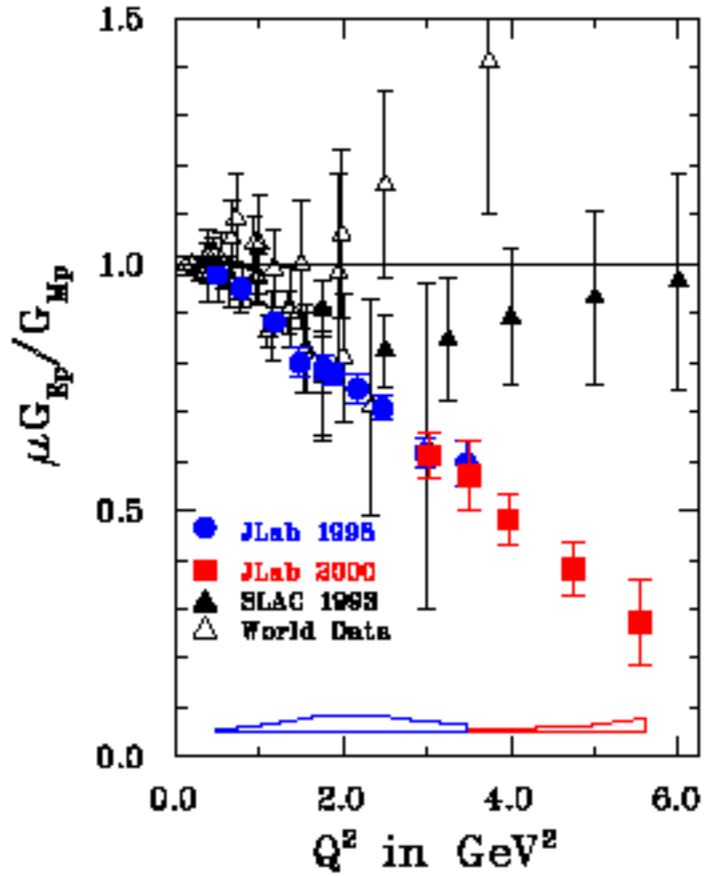
wf mainly concentrated in the low x region

- ➢ energy levels expressed analytically
- ➢ unperturbed wf given by the 1/x term
- ➢ major contribution to the helicity amplitudes

Good results due to semplicity

E. Santopinto, F. Iachello, M.Giannini, Eur. Phys. J. A **1**, 307 (1998)

The nucleon elastic form factors



- elastic scattering of polarized electrons on polarized protons

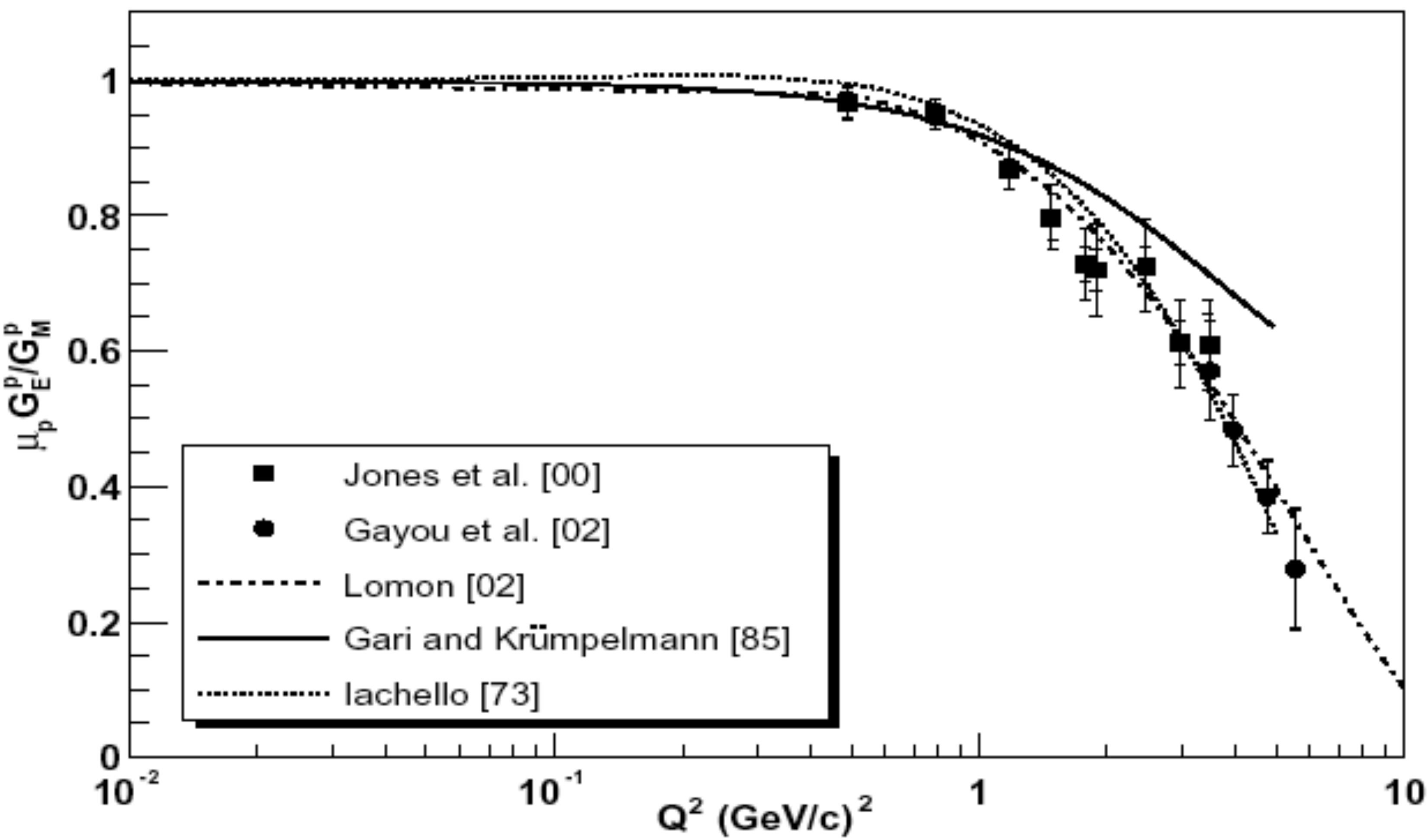
- measurement of polarizations asymmetry gives directly the ratio G^p_E/G^p_M

- discrepancy with Rosenbluth data (?)

- linear and strong decrease

- pointing towards a zero (!)

- latest data seem to confirm the behaviour



Jones et al. [00]
Gayou et al. [02]
Lomon [02]
Gari and Krümpelmann [85]
Iachello [73]
$\mu_p G_E^p / G_M^p$
Q^2 (GeV/c)^2
0
0.2
0.4
0.6
0.8
1
10^{-2}
10^{-1}
1
10

RELATIVITY

Various levels

- relativistic kinetic energy
- Lorentz boosts
- Relativistic dynamics
- quark-antiquark pair effects (meson cloud)
- relativistic equations (BS, DS)

Point Form Relativistic Dynamics

Point Form is one of the Relativistic Hamiltonian Dynamics
for a fixed number of particles (Dirac)

Construction of a representation of the Poincaré generators
P_μ (tetramomentum), J_k (angular momenta), K_i (boosts)
obeying the Poincaré group commutation relations
in particular

$$[P_k , K_i] = i\, \delta_{kj}\, H$$

Three forms:
Light (LF), Instant (IF), Point (PF)
Differ in the number and type of (interaction) free generators

Point form: P_μ interaction dependent

J_k and K_i free

Composition of angular momentum states as in the non relativistic case

Mass operator $M = M_0 + M_I$

$$M_0 = \Sigma_i \sqrt{\vec{\mathbf{p}}_i^{\,2} + m^2} \qquad \Sigma_i \vec{\mathbf{p}}_i = 0$$

$\vec{\mathbf{P}}_i$ undergo the same Wigner rotation -> M_0 is invariant

Similar reasoning for the hyperradius

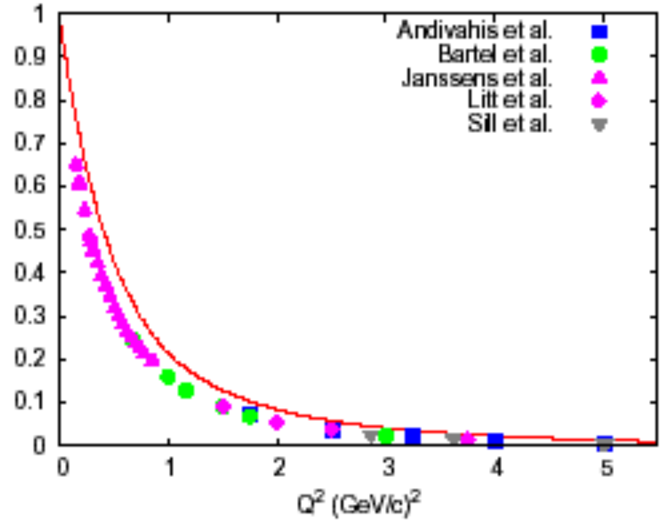
The eigenstates of the relativistic hCQM are interpreted as eigenstates of the mass operator M

Moving three-quark states are obtained through (interaction free) Lorentz boosts (velocity states)

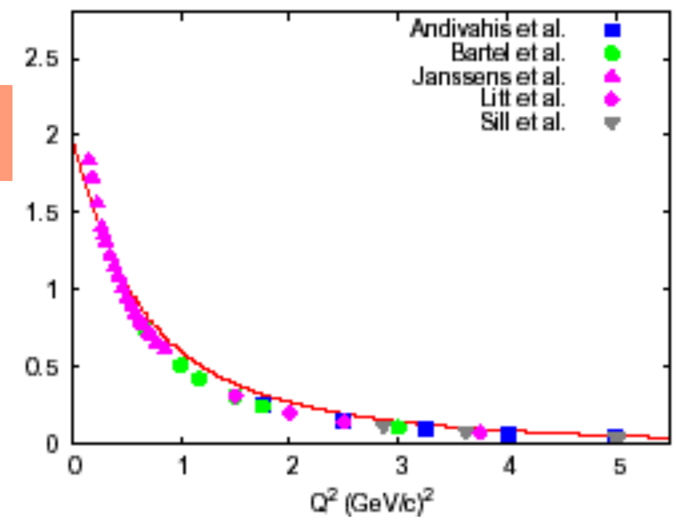
Calculated values!

- Boosts to initial and final states
- Expansion of current to any order
- Conserved current

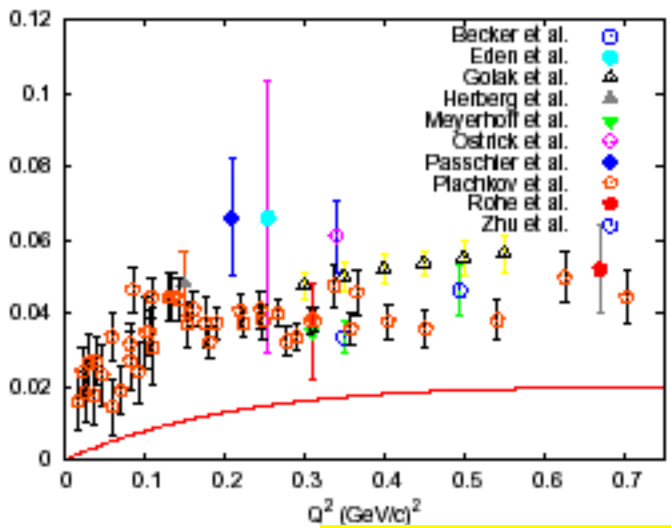
G_E^p



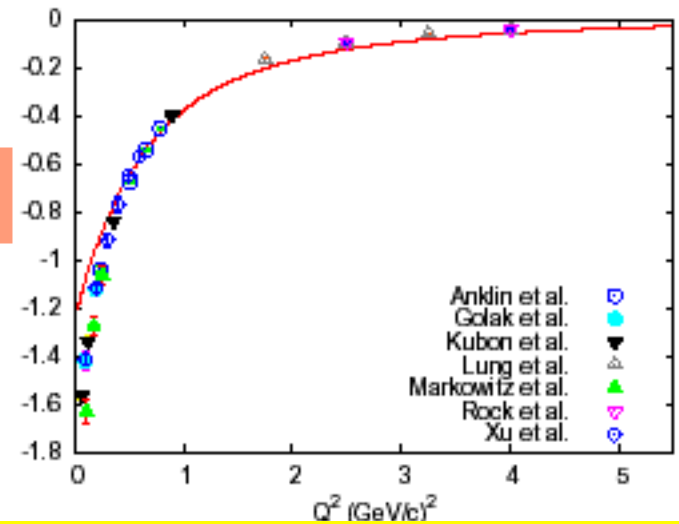
G_M^p



G_E^n

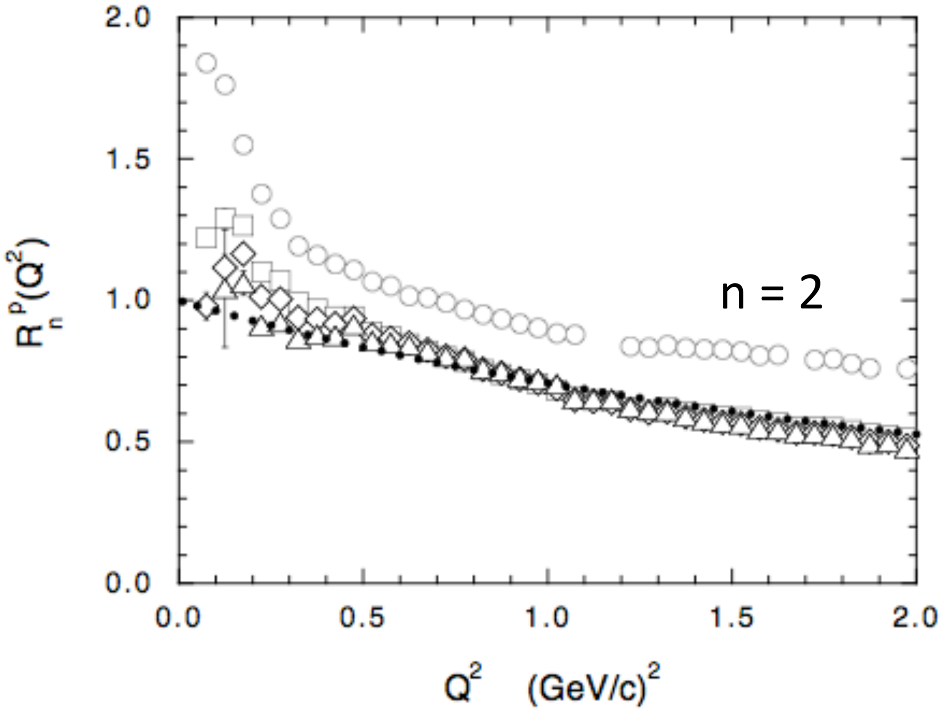


G_M^n



De Sanctis, Giannini, Santopintoi, Vassallo Phys. Rev. C76, 062201 (2007)

Further support 2



Ratio between
proton Nachtmann moments &
CQ distribution

Bloom-Gilman duality

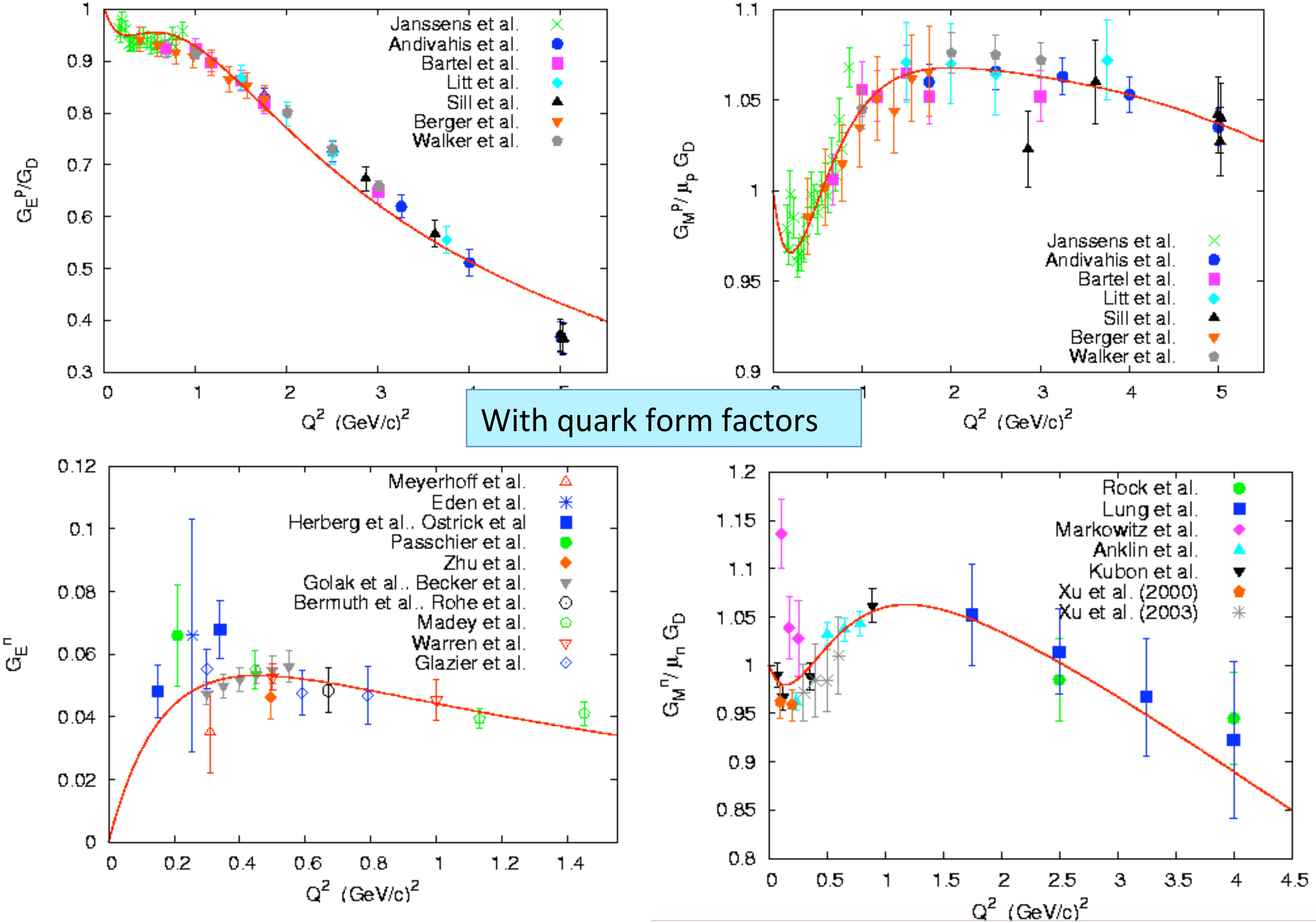
Inelastic proton scattering as elastic scattering on CQ

(approximate) scaling function → square of CQ ff

$F(Q^2) = 1/(1+ 1/6\, r_{CQ}^2\, Q^2)$

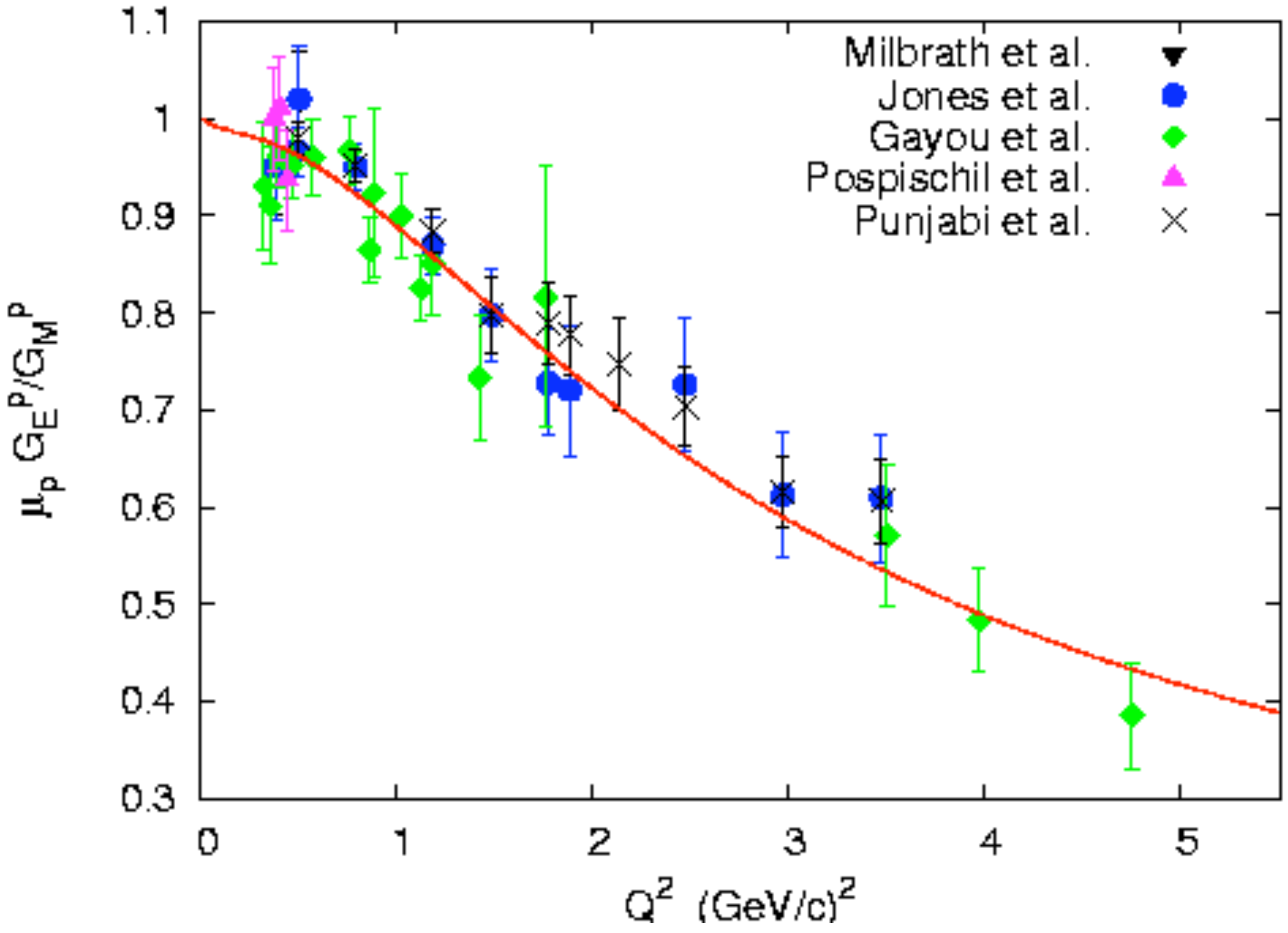
$r_{CQ} \cong$ **0.2-:0.4 fm**

Ricco ,,et al., PR **D67,** 094004 (2003)



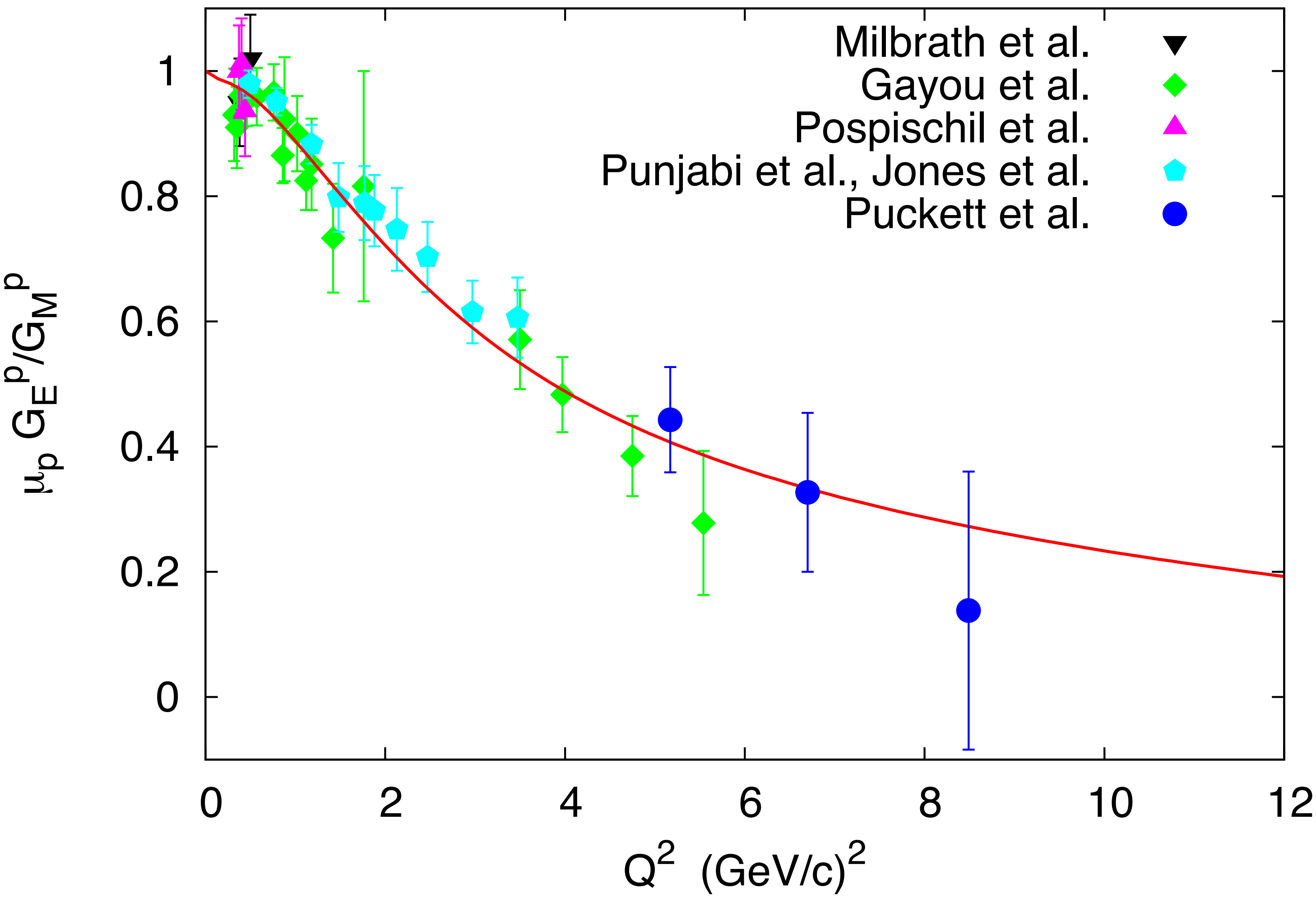
With quark form factors

De Sanctis, Giannini, Santopinto, Vassallo Phys. Rev. C76, 062201 (2007)

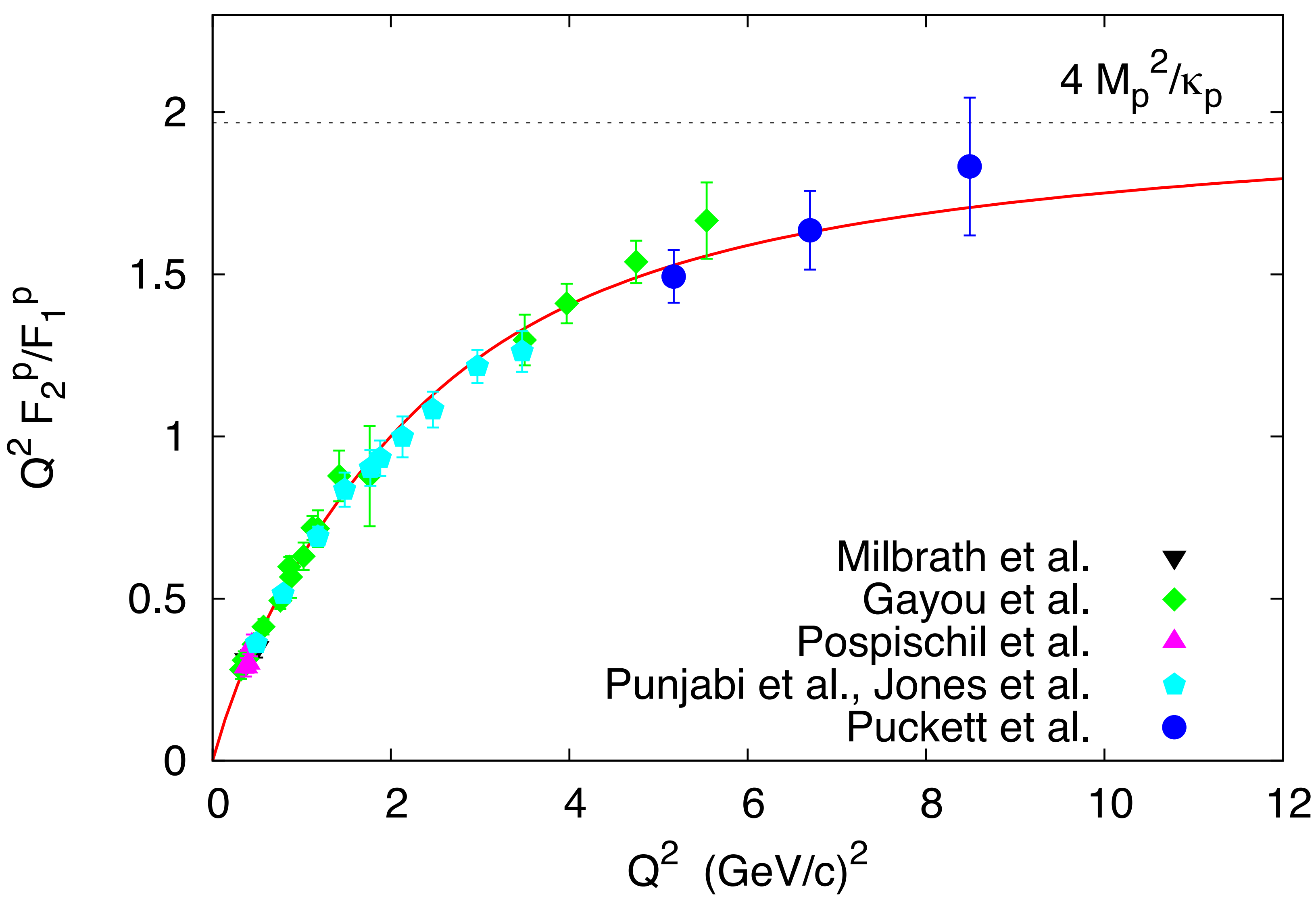


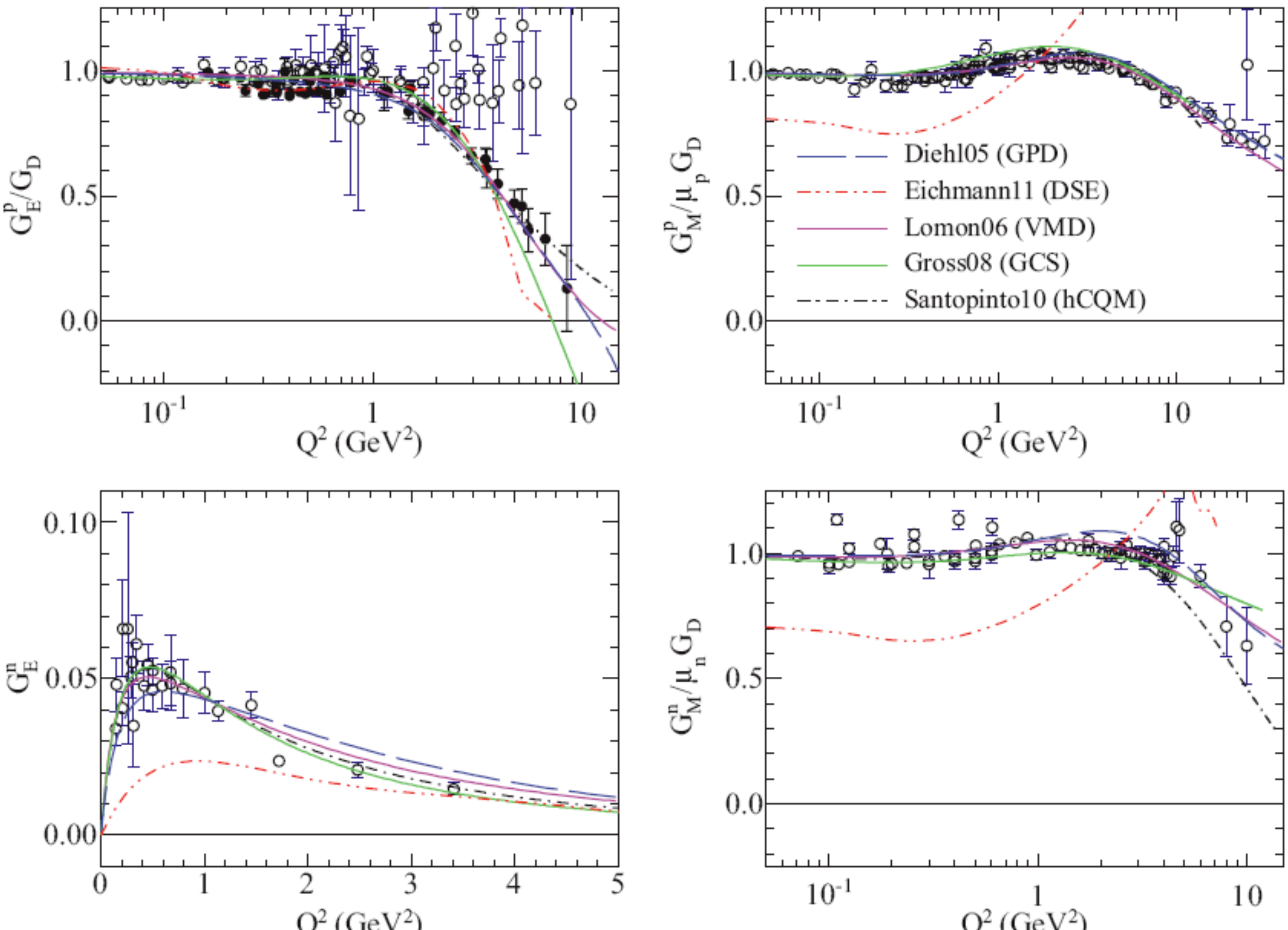
De Sanctis, Giannini, Santopinto, Vassallo Phys. Rev. C76, 062201 (2007)

E. Santopinto, A. Vassallo, M. M. Giannini, and M. De Sanctis, Phys. Rev. C 82, 065204 (2010)



E. Santopinto, A. Vassallo, M. M. Giannini, and M. De Sanctis, Phys. Rev. C 82, 065204 (2010)

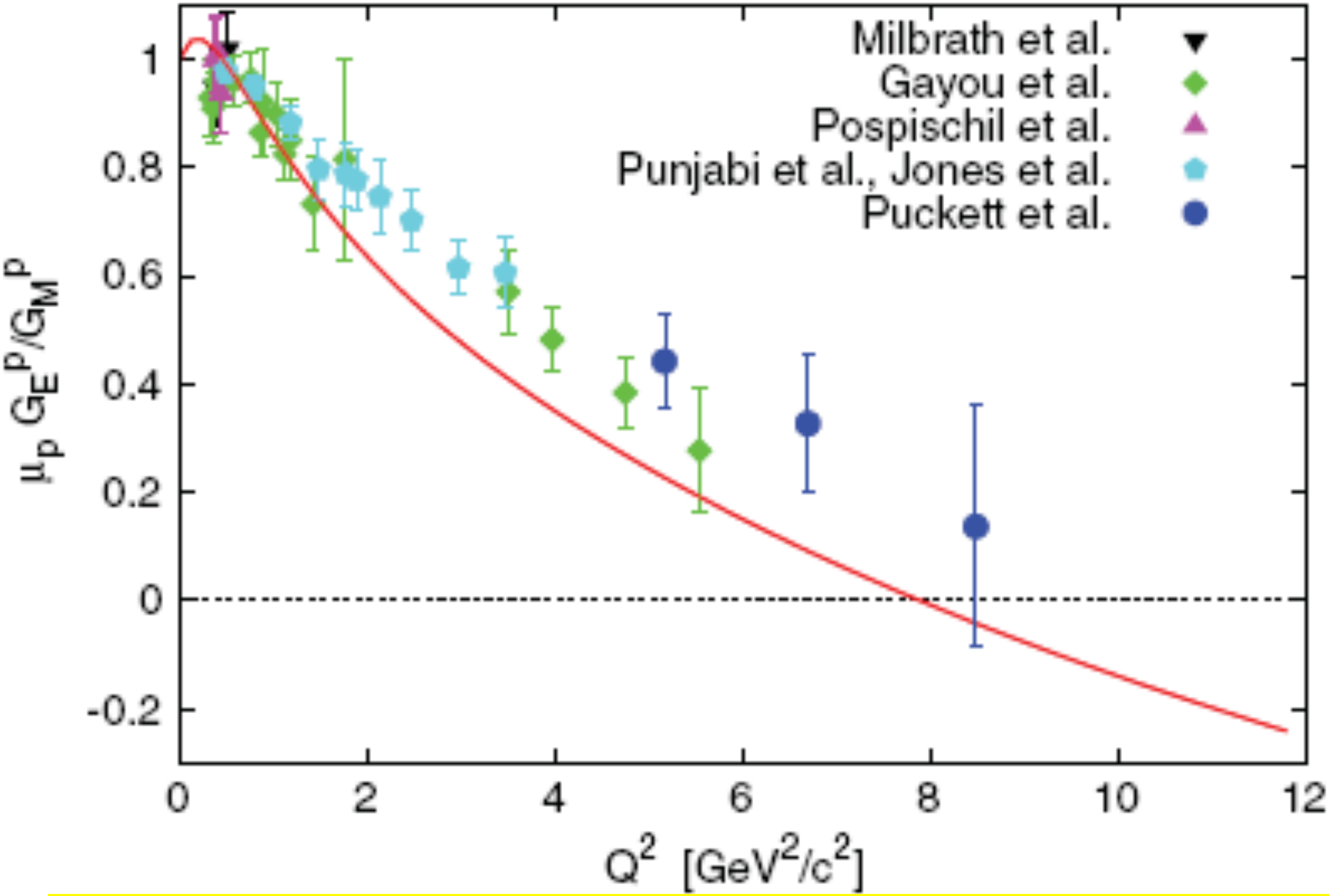




G_E^p/G_D
$G_M^p/\mu_p G_D$
G_E^n
$G_M^n/\mu_n G_D$
Q^2 (GeV^2)
Diehl05 (GPD)
Eichmann11 (DSE)
Lomon06 (VMD)
Gross08 (GCS)
Santopinto10 (hCQM)

Ratio $\mu_p\, G_E^p/G_M^p$

*De Sanctis, Ferretti, Santopinto, Vassallo, Phys. Rev. C **84**, 055201 (2011)*



Interacting Quark Diquark model , *E. Santopinto, Phys. Rev. C **72**, 022201(R) (2005)*

Unquenching the quark model

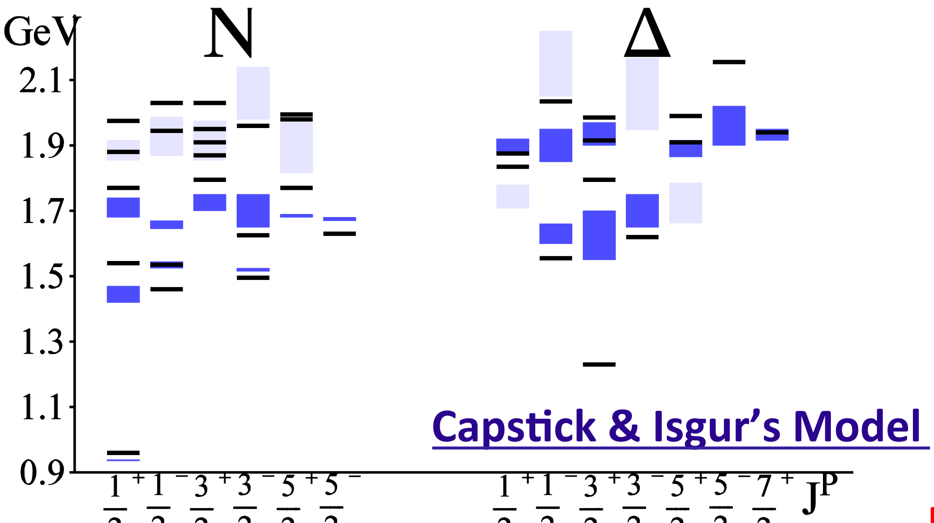
E. .Santopinto,. Bijker PRC 80, 065210 (2009),
PRC 82, 062202 (2010);J. Ferrettii,Santopinto, Bijker
Phys. Rev. C 85, 035204 (2012)

.

different CQMs for bayons

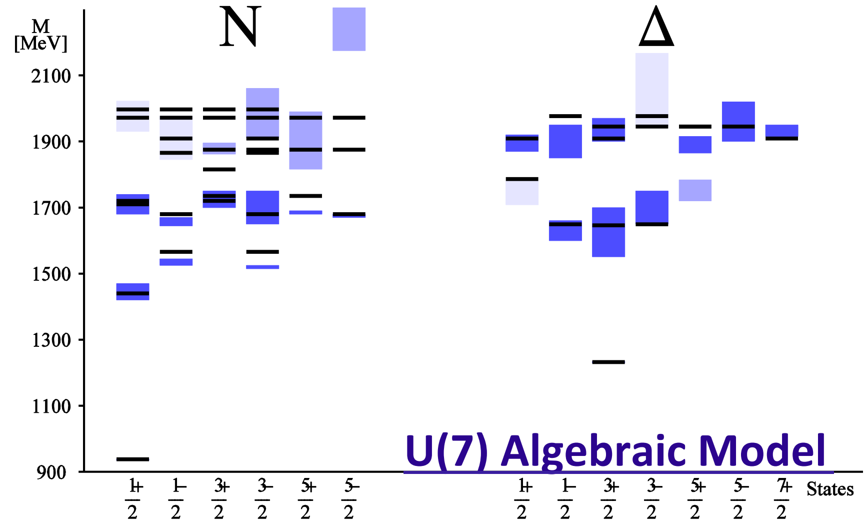
	Kin. Energy	SU(6) inv	SU(6) viol	date
Isgur-Karl	non rel	h.o. + shift	OGE	1978-9
Capstick-Isgur	rel	string + coul-like	OGE	1986
U(7) B.I.L.	rel M^2	vibr+L	Guersey-R	1994
Hyp. O(6)	non rel/rel	hyp.coul+linear	OGE	1995
Glozman Riska	non rel/rel**Plessas**	h.o./linear	GBE	1996
Bonn	rel	linear 3-body	instanton	2001

Non strange spectrum



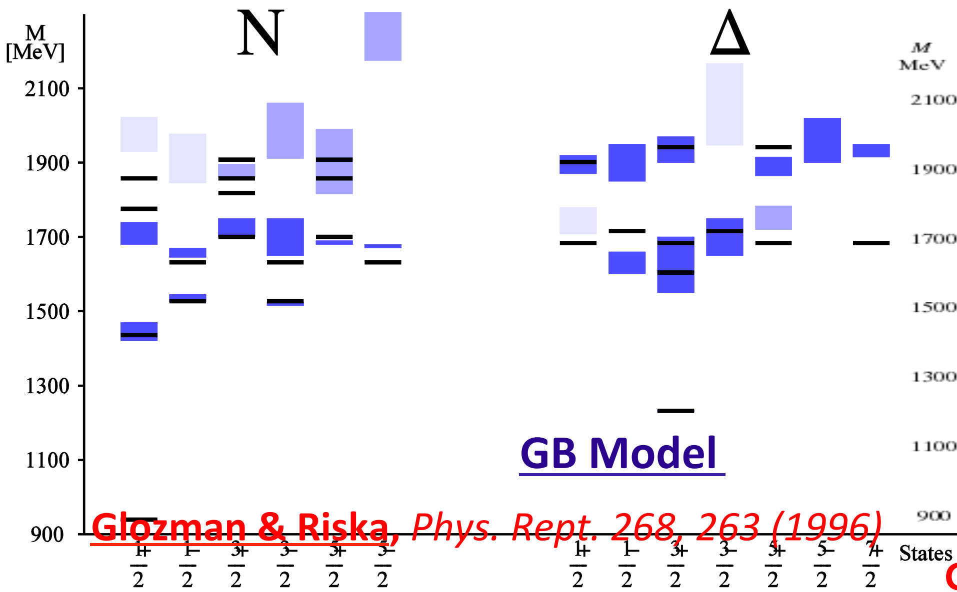
Capstick & Isgur's Model

Capstick and Isgur, *Phys. Rev. D34, 2809.*



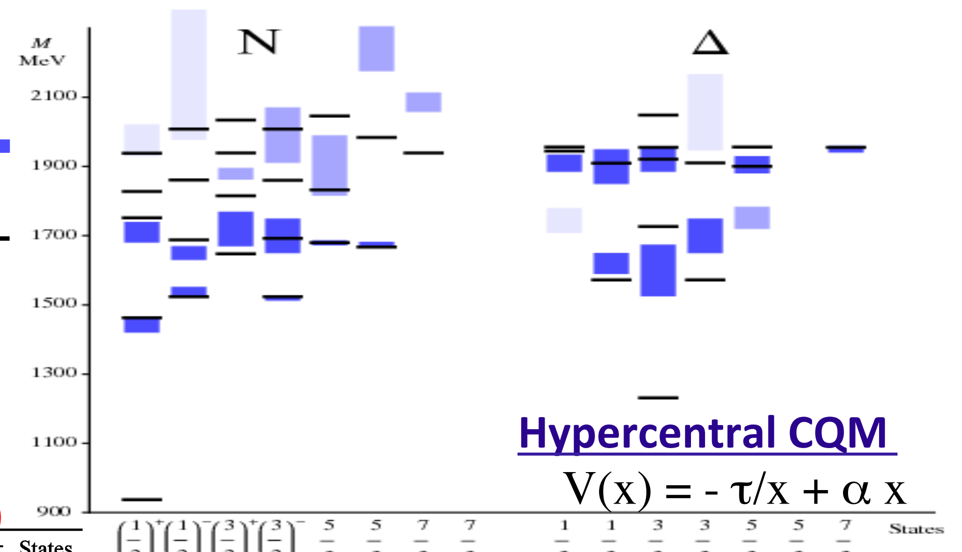
U(7) Algebraic Model

Bijker, Iachello, Leviatan, Ann. Phys. 236, 69 (1994)



GB Model

Glozman & Riska, *Phys. Rept. 268, 263 (1996)*



Hypercentral CQM

$V(x) = -\tau/x + \alpha x$

Giannini, Santopinto, Vassallo, *Eur. Phys. J .A12:447*

Many versions of CQMs have been developed

(IK, CI, GBE, U(7), hCQM,Bonn, etc.)

non relativistic and relativistic

While these models display peculiar features,

they share the following main features :

the effective degrees of freedom of 3q and a confining potential

the underling O(3) SU(3) symmetry

All of them are able to give a good description of the 3 and 4 stars spectrum

CQMs: S

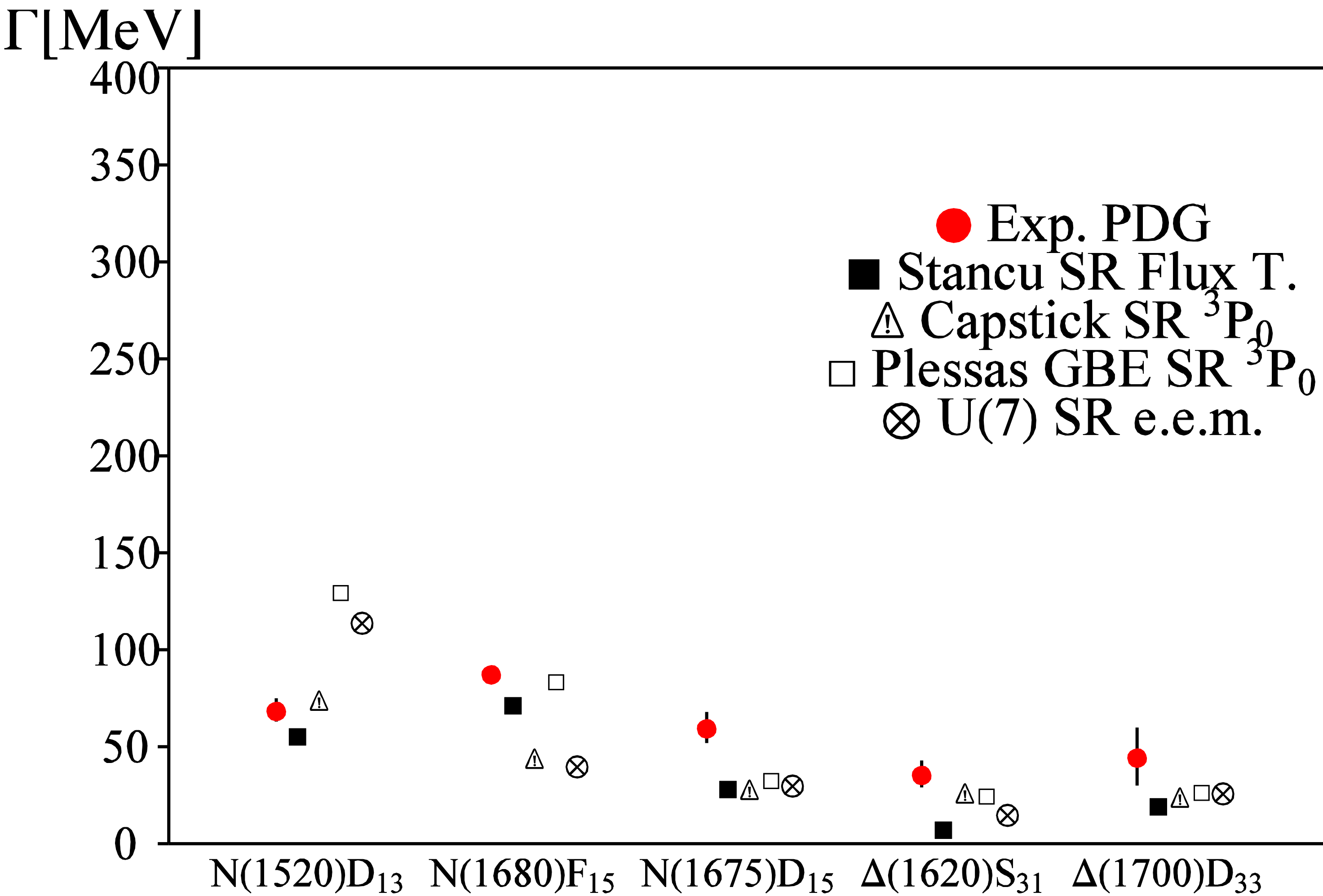
Good description of the spectrum and magnetic moments

Predictions of many quantities:

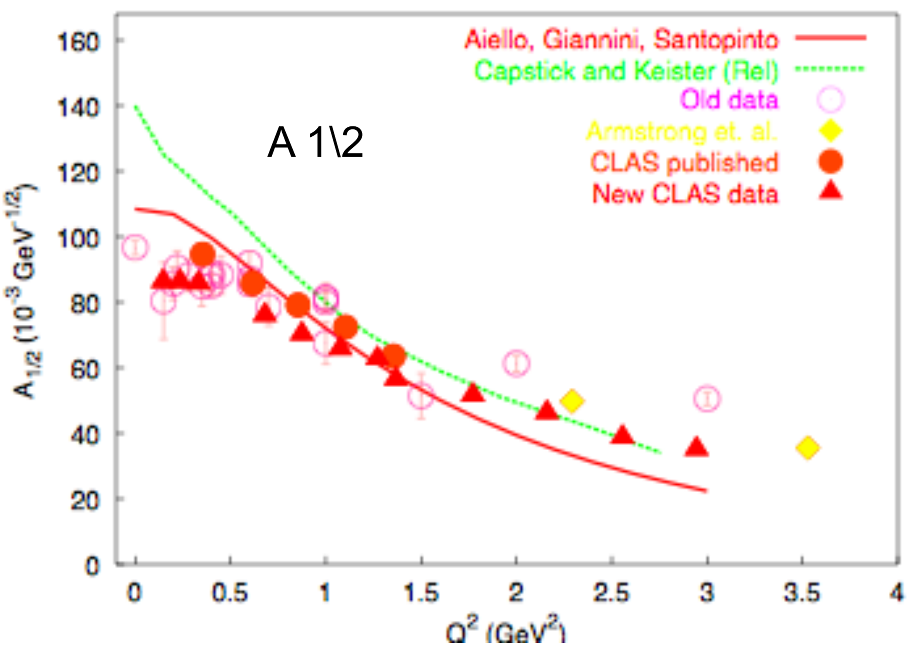
- strong couplins
- photocouplings
- helicity amplitudes
- elastic form factors
- structure functions

Based on the effective degrees of freedom of 3 constituent quarks

$\Gamma_{N\pi}$ width – Rel. Models



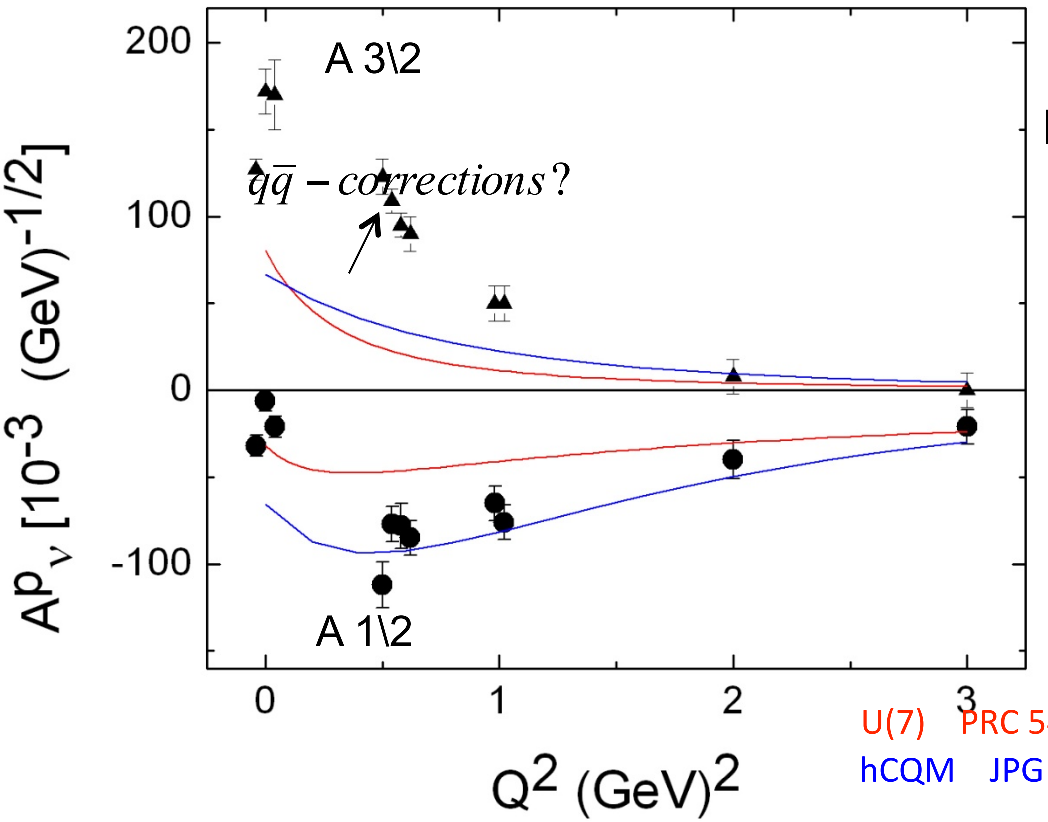
A_1/2(helicity amplitude ½) for the S11



hCQM JPG 24, 753 (1998)

Is it a degrees of freedom problem?

$q\bar{q}$ corrections ? important in the outer region



D13 transition amplitudes

U(7) PRC 54, 1935 (1996)

hCQM JPG 24, 753 (1998)

Considering also CQMs for mesons, CQMs able to reproduce the **overall trend of hundred of data**

- … but they show very similar deviations for observables such as

- photocouplings

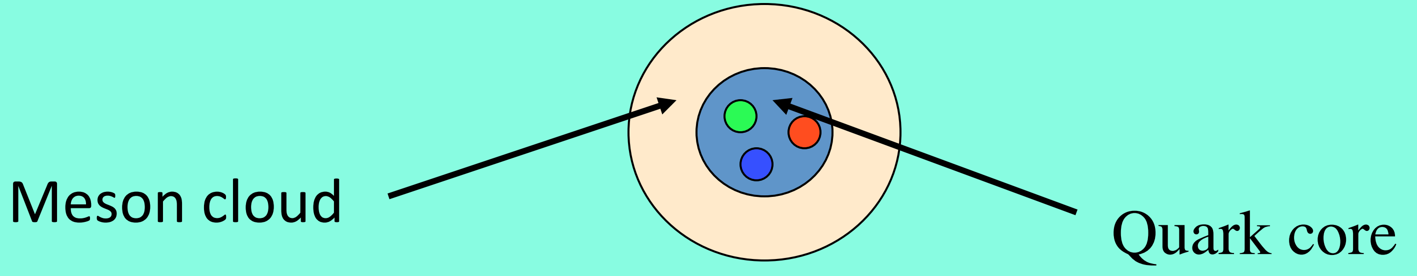
- helicity amplitudes,

please note

- the medium Q^2 behaviour is fairly well reproduced
- there is lack of strength at low Q^2 (outer region) in the e.m. transitions

- emerging picture:

  quark core plus (meson or sea-quark) **cloud**



There are two possibilities:

phenomenological parametrization

microscopic explicit quark description

Problems

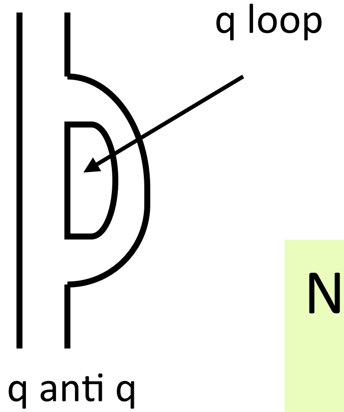
1) find a quark pair creation mechanism QCD inspired

2) implementation of this mechanism at the quark level but in such a way to

do not destroy the good CQMs results

Unquenching the quark model

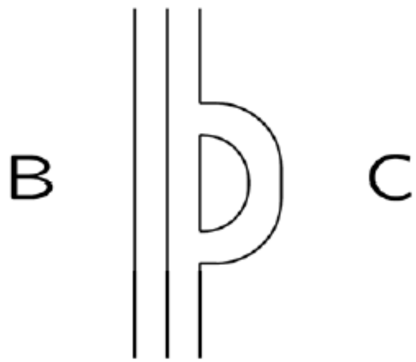
Mesons P. Geiger, N. Isgur, Phys. Rev. D41, 1595 (1990)
D44, 799 (1991)



Pair-creation operator with 3P0 quantum number

Note:
- sum over complete set of intermediate states necessary for preserving the OZI rule
- linear interaction is preserved after renormalization of the string constant

Unquenched Quark Model



Strange quark-antiquark pairs in the proton with h.o. wave functions

Tornqvist & Zenczykowski (1984)
Geiger & Isgur, PRD 55, 299 (1997)
Isgur, NPA 623, 37 (1997)

- Pair-creation operator with $^3P_0$ quantum numbers of vacuum

The good magnetic moment results of the CQM are preserved by the UCQM

Bijker, Santopinto,Phys.Rev.C80:065210,2009.

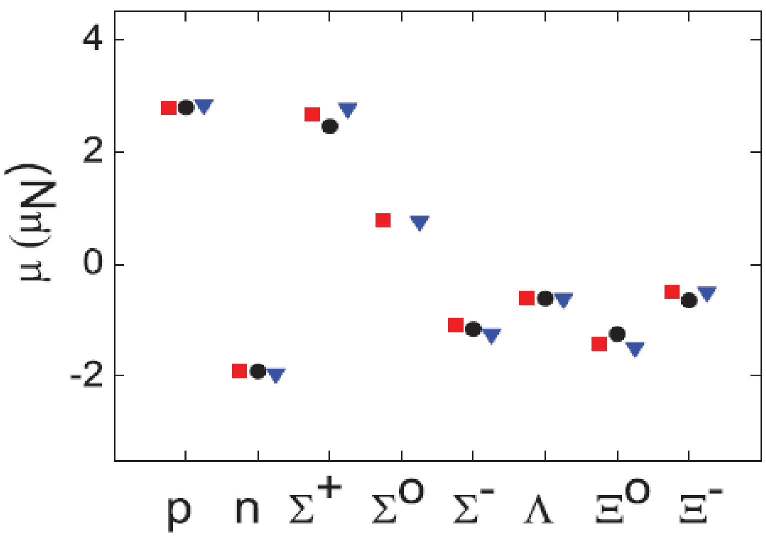
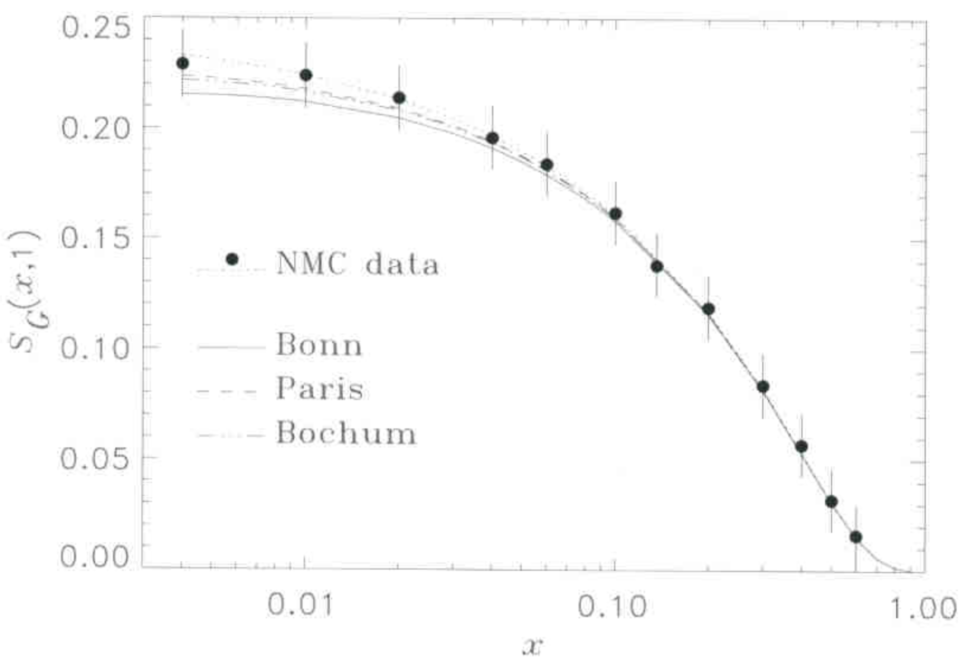


FIG. 3. (Color online) Magnetic moments of octet baryons: experimental values from the Particle Data Group [34] (circles), CQM (squares), and unquenched quark model (triangles).

Flavor Asymmetry

Gottfried sum rule

$$S_G = \int_0^1 dx \frac{F_{2p}(x) - F_{2n}(x)}{x} = \frac{1}{3} - \frac{2}{3}\int_0^1 dx \left[\bar{d}(x) - \bar{u}(x)\right]$$



$$S_G \neq \frac{1}{3} \Rightarrow N_{\bar{d}} \neq N_{\bar{u}}$$

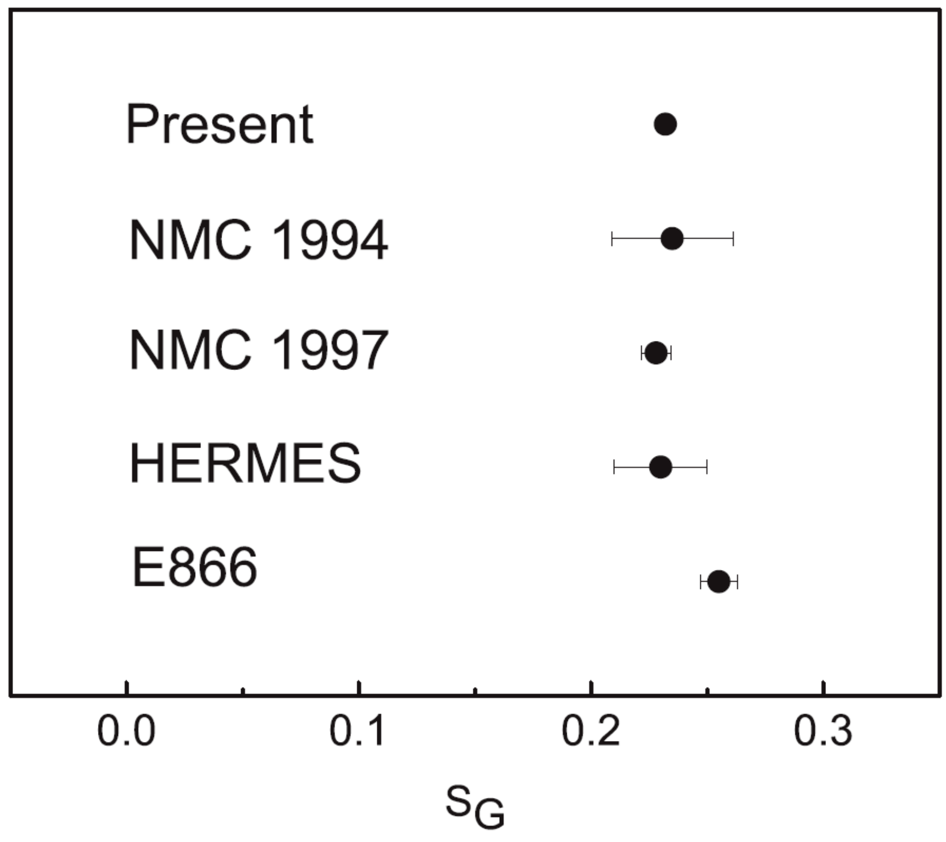
$$S_G = 0.2281 \pm 0.0065$$

$$\int_0^1 dx \left[\bar{d}(x) - \bar{u}(x)\right] = 0.16 \pm 0.01$$

$$S_G(x,1) = \int_x^1 dx' \frac{F_{2p}(x') - F_{2n}(x')}{x'}$$

Proton Flavor asymmetry

Santopinto, Bijker, PRC 82,062202(R) (2010)



Flavor asymmetry of the octect baryons in the UCQM

Santopinto, Bijker, PRC 82,062202(R) (2010)

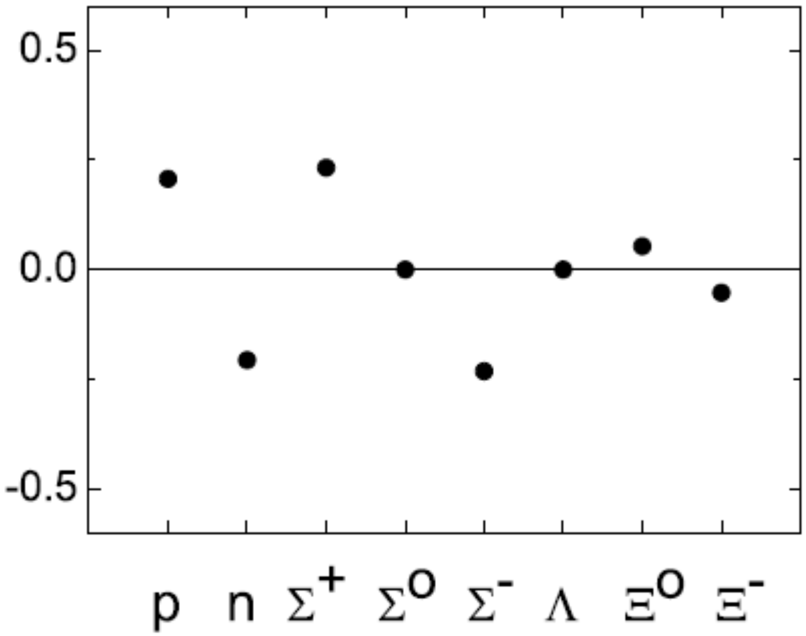


Figure 1. Flavor asymmetry of octet baryons

Flavor asymmetry
Nonperturbative QCD

Pauli blocking (Field & Feynman, 1977) too small
Pion dressing of the nucleon (Thomas et al., 1983)
Meson cloud models

Flavor asymmetries of octect baryons

Santopinto, Bijker, PRC 82,062202(R) (2010)

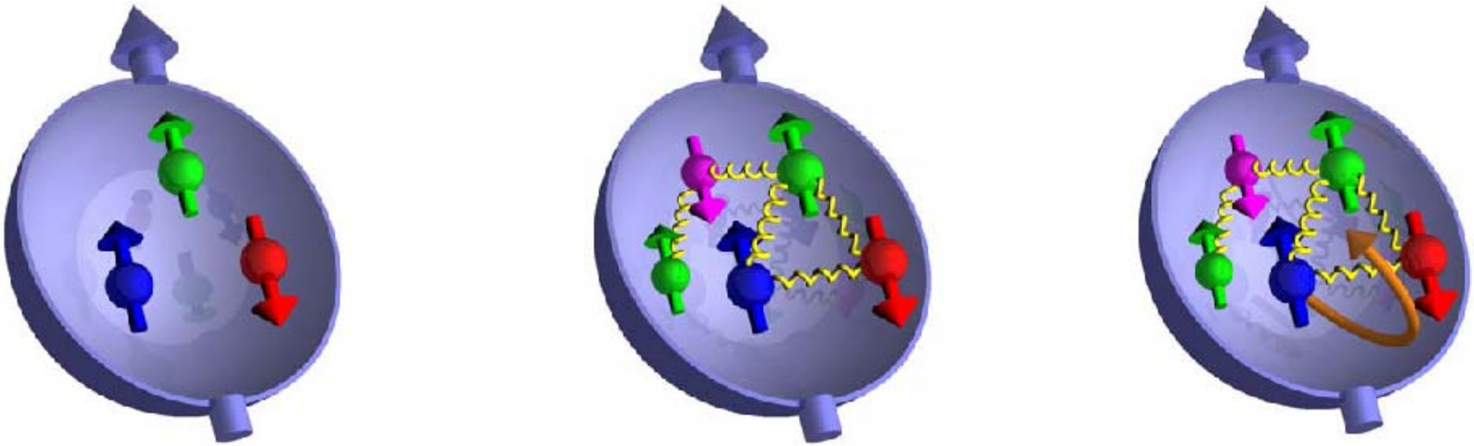
TABLE III. Relative flavor asymmetries of octet baryons.

Model	$\mathcal{A}(\Sigma^+)/\mathcal{A}(p)$	$\mathcal{A}(\Xi^0)/\mathcal{A}(p)$	Ref.
Unquenched CQM	0.833	−0.005	present
Chiral QM	2	1	Eichen
Balance model	3.083	2.075	Y.-J Zhang
Octet couplings	0.353	−0.647	Alberg

$$\Sigma^{\pm} p \to \ell^+ \ell^- + X \text{ (e.g., at CERN).}$$

3. Proton Spin Crisis

1980's | 1990's | 2000's



Naive parton model
3 valence quarks

QCD: contributions from
sea quarks and gluons

.. and orbital angular
momentum

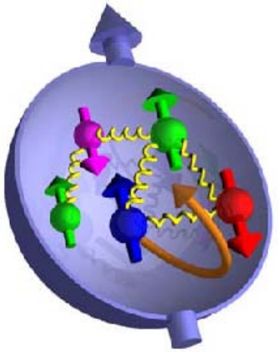
$$\frac{1}{2} = \frac{1}{2}(\Delta u + \Delta d)$$

$$\frac{1}{2} = \frac{1}{2}\underbrace{(\Delta u + \Delta d + \Delta s)}_{\Delta\Sigma} + \Delta G + \Delta L$$

$$\left.\begin{array}{rr} \Delta u = & 0.842 \\ \Delta d = & -0.427 \\ \Delta s = & -0.085 \end{array}\right\} \Delta\Sigma = 0.330 \pm 0.039$$

HERMES, PRD 75, 012007 (2007)
COMPASS, PLB 647, 8 (2007)

Proton Spin



- COMPASS@CERN: Gluon contribution is small (sign undetermined)
- Unquenched quark model

Ageev et al., PLB 633, 25 (2006)
Platchkov, NPA 790, 58 (2007)

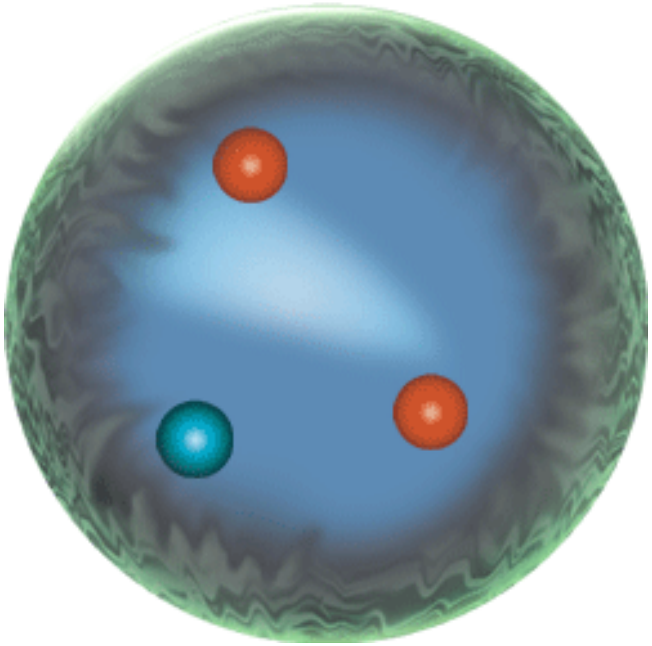
		CQM	Unquenched QM		
			Valence	Sea	Total
p	$\Delta\Sigma$	1	0.378	0.298	0.676
	$2\Delta L$	0	0.000	0.324	0.324
	$2\Delta J$	1	0.378	0.622	1.000

- More than half of the proton spin from the sea!
- Orbital angular momentum

Suggested by Myhrer & Thomas, 2008, but not explicitly calculated

4. Strangeness in the Proton

- The strange (anti)quarks come uniquely from the sea: there is no contamination from up or down valence quarks
- The strangeness distribution is a very sensitive probe of the nucleon's properties
- Flavor content of form factors
- New data from Parity Violating Electron Scattering experiments: SAMPLE, HAPPEX, PVA4 and G0 Collaborations



"There is no excellent beauty that hath not some strangeness in the proportion"
(Francis Bacon, 1561-1626)

Quark Form Factors

- Charge symmetry

$$G^{u,p} = G^{d,n} \equiv G^{u}$$
$$G^{d,p} = G^{u,n} \equiv G^{d}$$
$$G^{s,p} = G^{s,n} \equiv G^{s}$$

- Quark form factors

$$G^{u} = \left(3 - 4\sin^2\Theta_W\right) G^{\gamma,p} - G^{Z,p}$$
$$G^{d} = \left(2 - 4\sin^2\Theta_W\right) G^{\gamma,p} + G^{\gamma,n} - G^{Z,p}$$
$$G^{s} = \left(1 - 4\sin^2\Theta_W\right) G^{\gamma,p} - G^{\gamma,n} - G^{Z,p}$$

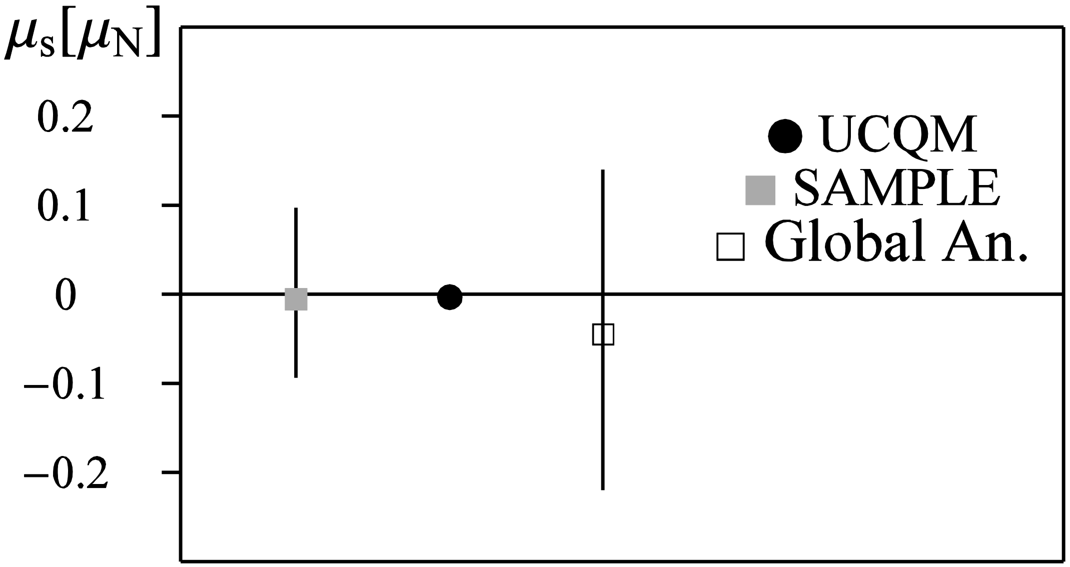
Kaplan & Manohar, NPB 310, 527 (1988)
Musolf et al, Phys. Rep. 239, 1 (1994)

Static Properties

$G_E(0) = e$	Electric charge
$G_M(0) = \mu$	Magnetic moment
$\left\langle r^2 \right\rangle_E = -6 \left. \frac{dG_E}{dQ^2} \right\|_{Q^2=0}$	Charge radius
$\left\langle r^2 \right\rangle_M = -\frac{6}{\mu} \left. \frac{dG_M}{dQ^2} \right\|_{Q^2=0}$	Magnetic radius

Strange Magnetic Moment

$$\vec{\mu}_s = \sum_i \mu_{i,s} \left[2\vec{s}(q_i) + \vec{\ell}(q_i) - 2\vec{s}(\bar{q}_i) - \vec{\ell}(\bar{q}_i) \right]$$

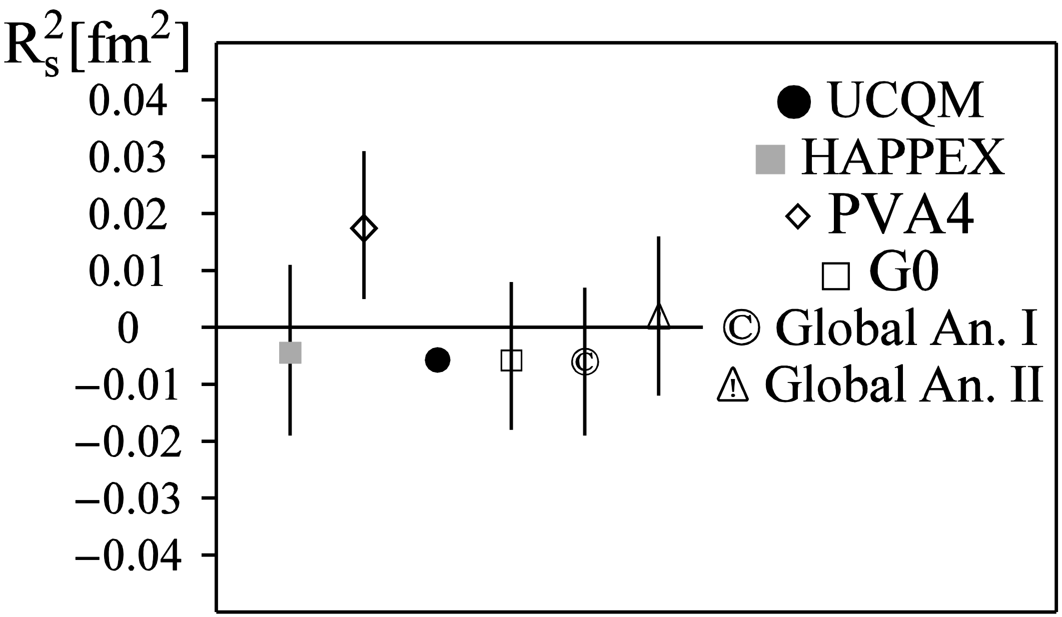


Jacopo Ferretti, Ph.D. Thesis, 2011

Bijker, Ferretti, Santopinto, Phys. Rev. C **85, 035204 (2012)**

Strange Radius

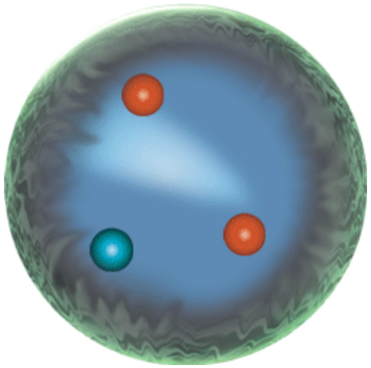
$$R_s^2 = \sum_{i=1}^{5} e_{i,s} \left(\vec{r}_i - \vec{R}_{CM}\right)^2$$



Jacopo Ferretti, Ph.D. Thesis, 2011

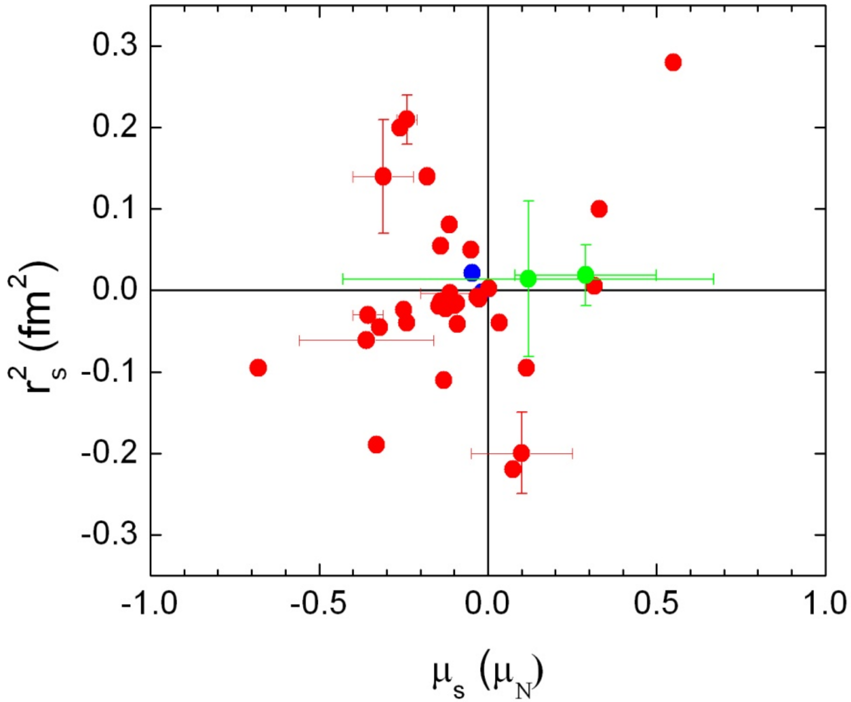
Bijker, Ferretti, Santopinto, Phys. Rev. C **85, 035204 (2012)**

Strange Proton



- Strange radius and magnetic moment of the proton
- Theory
- Lattice QCD
- Global analysis PVES
- Unquenched QM

$$\mu_s = -6 \cdot 10^{-4}\,(\mu_N)$$
$$\langle r^2 \rangle_s = -4 \cdot 10^{-3}\,(\mathrm{fm}^2)$$



Jacopo Ferretti, Ph.D. Thesis, 2011

Bijker, Ferretti, Santopinto, Phys. Rev. C **85, 035204 (2012)**

Main points

- Unquenching quark model:we have constructed the formalism in an explicit way, also thanks to group theory tecniques. Now, it can be applied to any quark model.
- We think we have maked up the problems of quark models adding the coupling with the continuum, thus opening the possibilty of many, many applications

- Future: application to open problems in hadron structure and spectroscopy : helicity amplitudes, strong decays,and so on.

Axial form factor of the nucleon

Adamuscìn,E. A. Kurae, Tomasi-G.,Maas,Phys. Rev. C 78, 035201 (2008)

Adamuscìn, Tomas-G.i, Santopinto, Bijker,Phys. Rev. C 78, 035201 (2008)

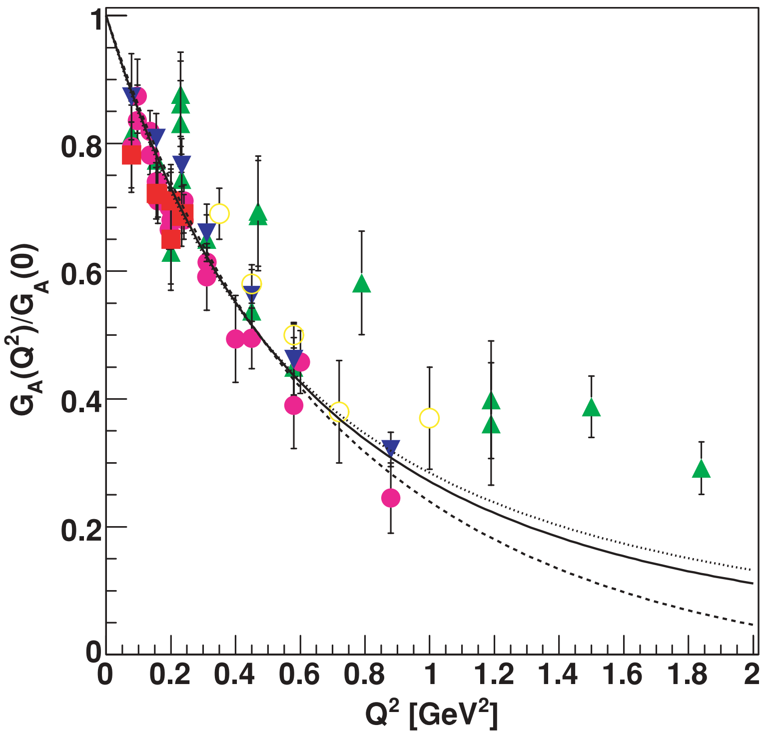


FIG. 1. (Color online) Comparison between theoretical and experimental values of the axial form factor of the nucleon $G_A(Q^2)$ as a function of Q^2. The theoretical values are calculated in the two-component model using Eq. (7) with $\alpha = 1.57$ and $\gamma = 0.25$ GeV^{-2} [12] (dashed line), and $\alpha = 0.95$ and $\gamma = 0.515$ GeV^{-2} [13] (solid line), and the dipole form of Eq. (1) with $M_A = 1.069$ GeV (dotted line). The experimental values were extracted according different

Adamuscìn, Tomasi, Santopinto, Bijker, Phys. Rev. C 78, 035201 (2008)

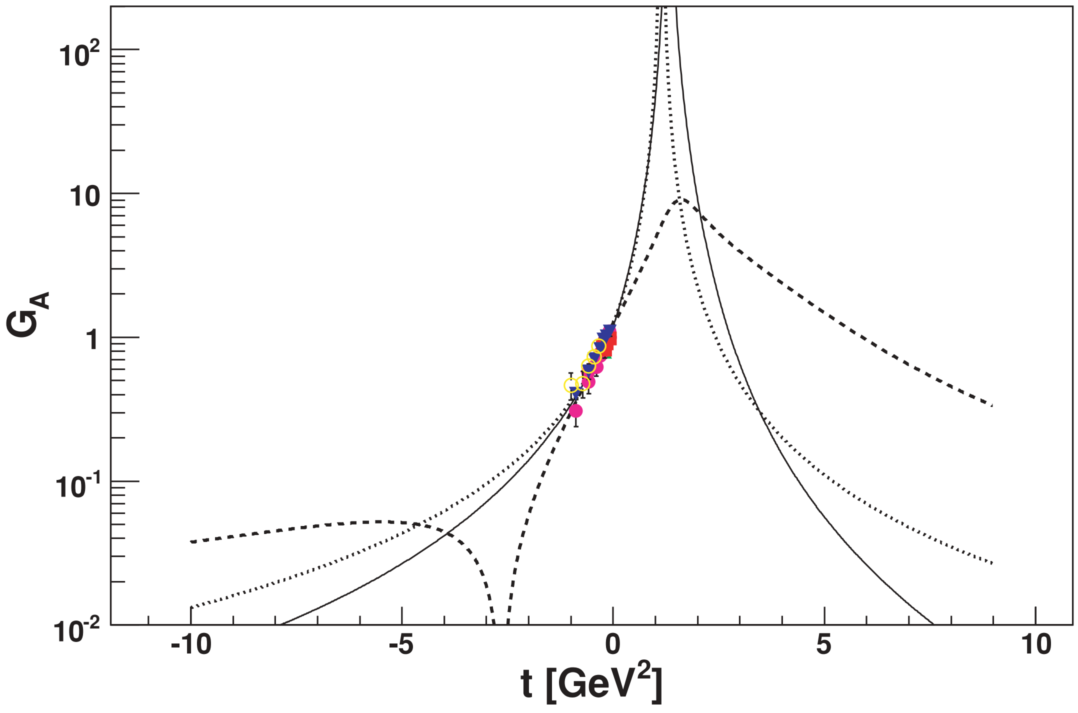


FIG. 2. (Color online) As Fig. 1, but for the absolute value of the axial form factor $|G_A(t)|$ in the space-like ($t < 0$) and time-like ($t > 0$) regions. In the time-like region, $\delta = 0.925$ for IJL [14] and $\delta = 0.397$ for BI [13].

III. TWO-COMPONENT MODEL

In the two-component model [12,13], the axial nucleon form factor is described as

$$G_A(Q^2) = G_A(0) g(Q^2) \left[1 - \alpha + \alpha \frac{m_A^2}{m_A^2 + Q^2} \right], \qquad g(Q^2) = \left(1 + \gamma Q^2\right)^{-2}, \tag{2}$$

with $Q^2 > 0$ in the space-like region. $g(Q^2)$ denotes the coupling to the intrinsic structure (three valence quarks) of the nucleon, and m_A is the mass of the lowest axial meson $a_1(1260)$ with quantum numbers $I^G(J^{PC}) = 1^-(1^{++})$ and $m_A = 1.230$ GeV. We note that, unlike other studies, in which m_A is a parameter, here it corresponds to the mass of the axial meson $a_1(1260)$. In the present case, γ is taken from previous studies of the electromagnetic form factors of the nucleon [12,13]. Therefore, α is the only fitting parameter.

Time like region

$$G_A(t) = G_A(0)g(t)\left[1 - \alpha + \alpha \frac{m_A^2\left(m_A^2 - t + i m_A \Gamma_A\right)}{\left(m_A^2 - t\right)^2 + (m_A \Gamma_A)^2}\right],$$

with

$$g(t) = \left(1 - e^{i\delta}\gamma t\right)^{-2}.$$

It can be measured with Experiments at FAIR and BES3