Corso di laurea in Fisica
A.A. 2002-2003

Tecniche Diagnostiche

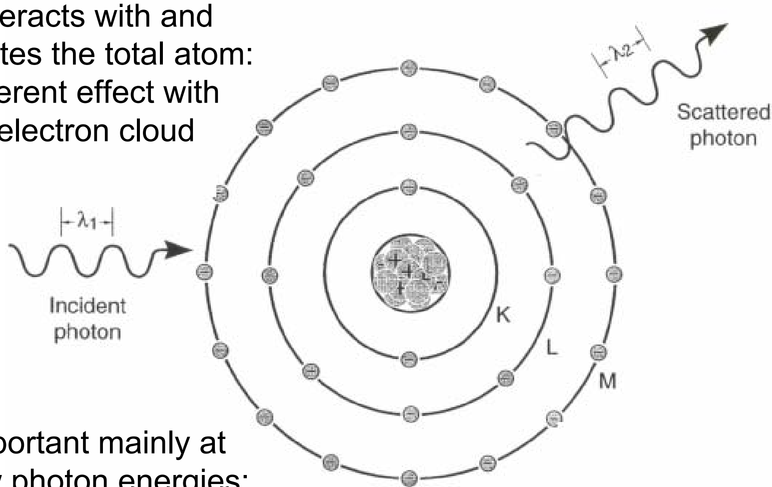
2 - Interazione radiazione-materia

Photon Interactions

- "Big-four" interactions:
  - **Rayleigh scattering**
  - **Compton scattering**
  - **Photoelectric Absorption**
  - **Pair Production**
- All except pair production play a role in diagnostic radiology and nuclear medicine

Rayleigh Scattering

γ interacts with and excites the total atom: coherent effect with the electron cloud

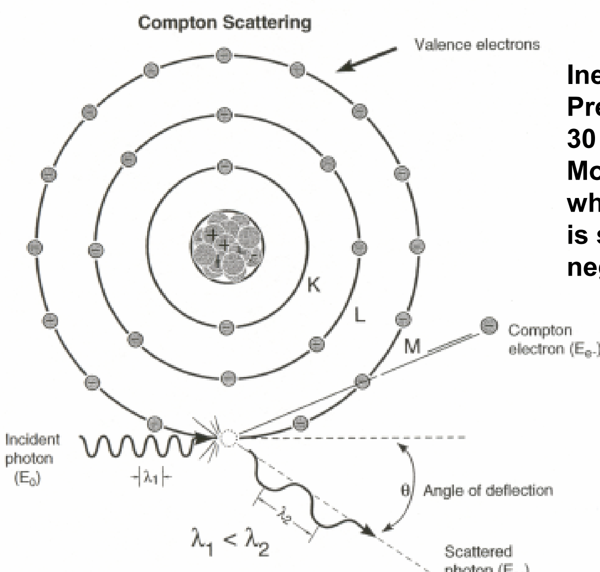


Important mainly at low photon energies: 15-30 keV: <10% of interactions

$\boxed{\lambda_1 = \lambda_2}$

No ionization: electrons not ejected

Compton Scattering (1)



Inelastic scattering
Predominant interaction: 30 keV – 30 MeV
Most likely with outer e, where the binding energy is small (eV) and can be neglected below:

$$E_0 = E_{sc} + E_e$$

Atom is ionized

Compton Scattering (2)

$$E_{sc} = \frac{E_0}{1 + \frac{E_0}{m_e c^2}\left(1 - \cos\theta\right)}$$

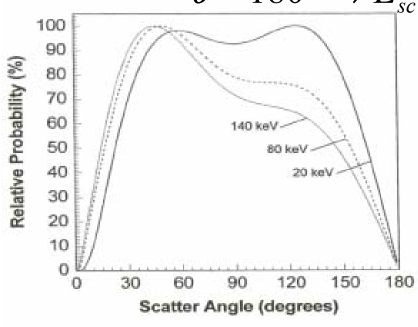
$m_e c^2$ = 511 keV

$$\theta = \theta_0 \rightarrow E_{sc} = E_0$$

no scattering

$$\theta = 180° \rightarrow E_{sc} = \frac{E_0}{1 + \frac{2E_0}{m_e c^2}}$$

back scattering



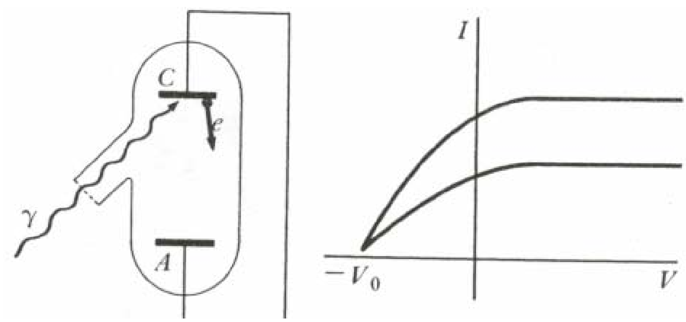
As the photon energy increases, scattered photons and electrons tend to scatter more in the forward direction. Such photons are likely to be detected in the image receptor and will reduce image contrast

Compton Scattering (3)

- At higher γ-energies, the majority of the energy is transferred to the scattered electron
- For 18-150 keV range, the majority of the energy is retained by the photon (80 keV photon after scattering has a minimum of 61 keV !)
- Interaction probability ~ electron density (N_e/g)
- Electron density is fairly constant as a function of Z, so the probability of scattering per unit volume is approx. proportional to density

The Photoelectric Effect (1)

- All incident photon energy is transferred to an electron, which is then ejected from an atom
- Discovered by Lenard in 1900



Two different light intensities

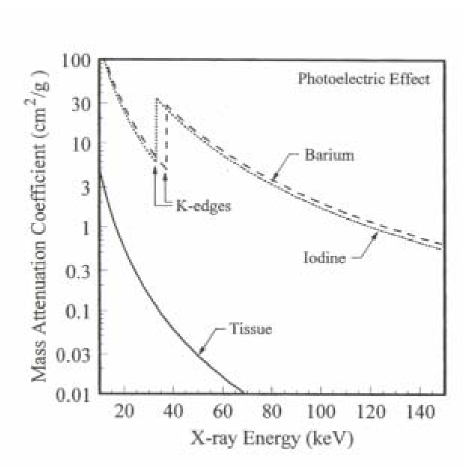
The Photoelectric Effect (2)

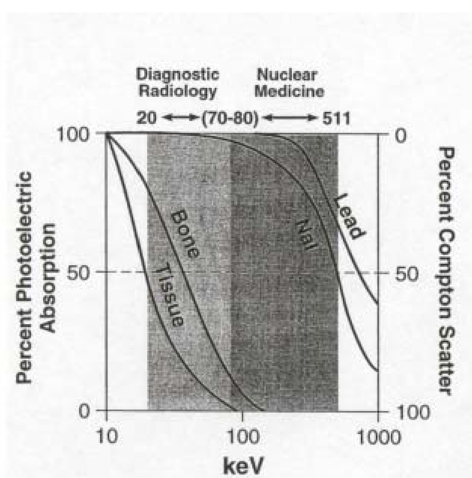
- V>0 electrons attracted to the anode. If V>>0 then all reach the anode and current is maximal. The max. current ~ light intensity
- V<0 electrons repelled from the anode. Only electrons with initial kinetic energy ½ mv2 >e|V| reach the anode. V0 is the stopping potential
- 1905 – Einstein explains this effect by stating that light consists of discrete quanta of energy (hn) and when one of these quanta reaches the cathode all of its energy is given to an electron.

The Photoelectric Effect (3)

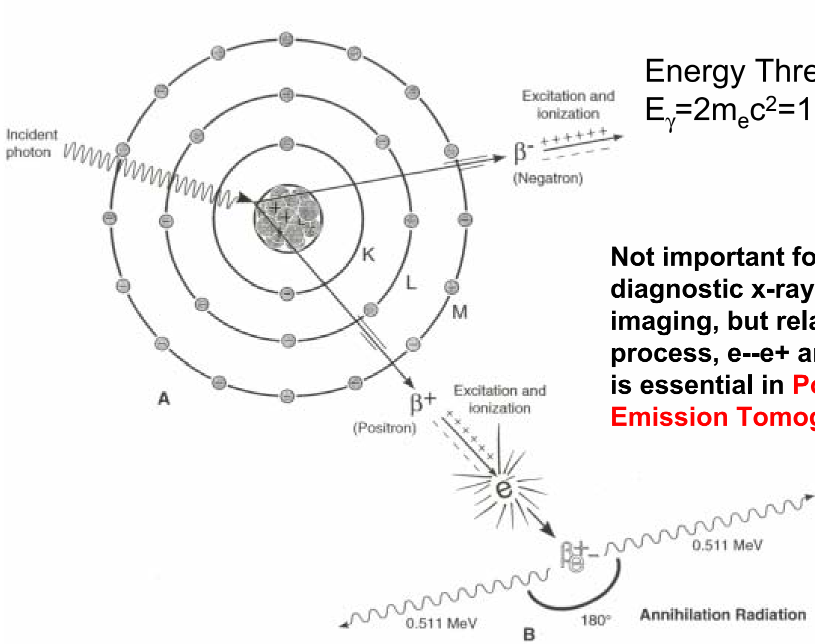
- The probability for this process jumps ("absorption edge") when the photon energy just exceeds the binding energy of inner shell electrons.
- The process is very significant for high-Z material (lots of electrons) such as Iodine and Barium…($\sim Z_3$)
- The probability for photoelectric absorption strongly depends on energy $\sim 1/E^3$

The Photoelectric Effect (4)





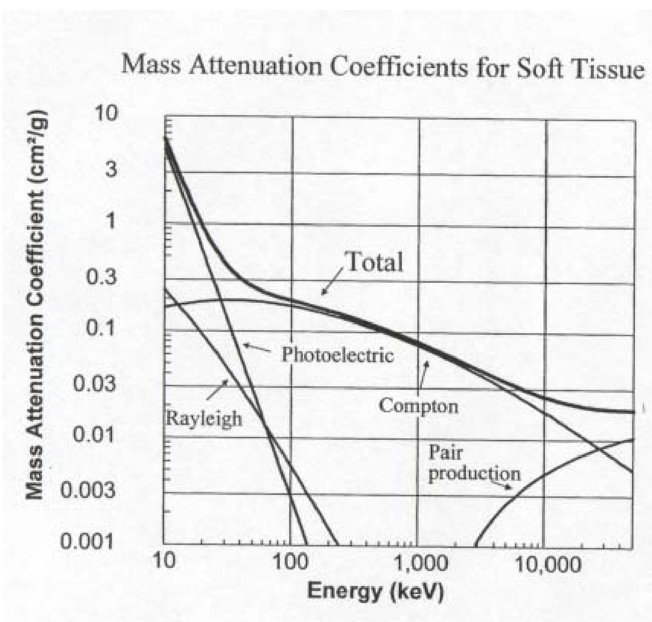
Pair production



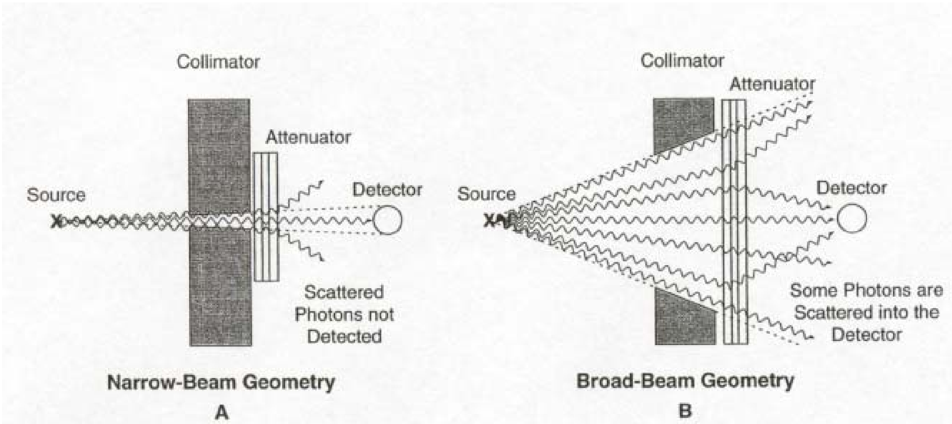
Energy Threshold:
$E_{\gamma}=2m_ec^2=1.02$ MeV

Not important for diagnostic x-ray imaging, but related process, e--e+ annihilation is essential in Positron Emission Tomography

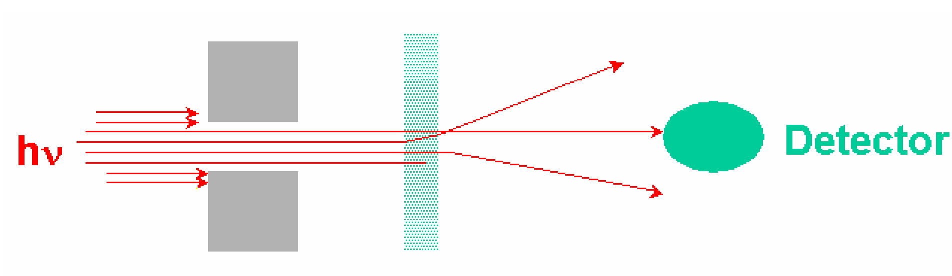
Attenuation of x- and γ rays



Attenuation of x- and γ rays



The Total Absorption Coefficient and Photon Attenuation



The loss of photons in the initial beam direction is called *attenuation* of the beam.

The Total Absorption Coefficient and Photon Attenuation

The total cross section per atom is:
$\sigma = \phi_{photon} + Z_e \sigma_c + \tau_{pair}$

The total interaction probability per photon in a unit traveling length is:
$\mu = N\sigma = \sigma(N_a\rho/A)$ (linear absorption coefficient)
N_a: Avogadro's Number;
ρ: density of the material;
A: Atomic weight

<u>Assumptions:</u>
A uniform incident beam
Homogeneous scattering

For compounds and mixtures:

$$\frac{\mu}{\rho} = w_1 \frac{\mu_1}{\rho_1} + w_1 \frac{\mu_2}{\rho_2} + \ldots$$

w_i is the weight fraction of the i-th component

The Total Absorption Coefficient and Photon Attenuation

Consider n incident photons and each one undergoes interaction independently (equal probability), then n photons are equivalent to n tries. The average number of total interactions is:

$$x = np$$

p: individual probability

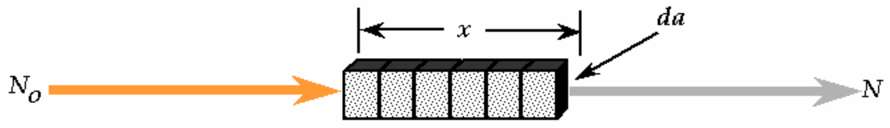
Therefore the attenuation of photons in distance $d\ell$ is :

$$\boxed{\begin{aligned} dn &= -n N\sigma\, d\ell = -n\mu\, d\ell \\ N(\ell) &= N_0 e^{-\int \mu\, d\ell} \end{aligned}}$$

$$\boxed{\mu = N\sigma = \sigma(N_a\rho/A)\ (\mathrm{cm}^{-1})}$$

μ is normally small $< 10^{-6}$ (m^{-1})

Homogeneous Attenuation



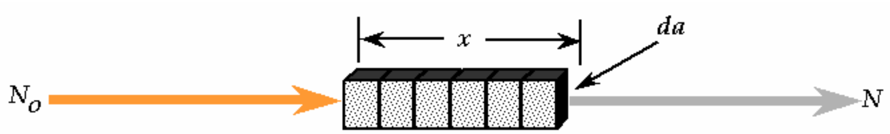
$$\int \frac{dN}{N} = -\int \mu_z(x)\,dx$$

$$N = N_0 e^{-\int \mu_z(x)dx}$$

$$\ln \frac{N}{N_0} = \int \mu_z(x)\,dx = \mu x$$

$$N = N_0 e^{-\mu x}$$

Heterogeneous Attenuation



$$\int \frac{dN}{N} = -\int \mu_z(x)\,dx$$

$$N = N_0 e^{-\int \mu_z(x)dx}$$

$$\ln \frac{N}{N_0} = \int \mu_z(x)\,dx = \sum_i \mu_i \Delta x$$

Mass Attenuation Coefficient

$\mu = N\sigma = \sigma\,(N_a\rho/A)$

mass attenuation coefficient:

$$\boxed{\mu/\rho = \sigma\,(N_a/A) \quad (m^2kg^{-1})}$$

$$\boxed{N(x) = N_0 e^{-(\mu/\rho)\rho x}}$$

$$\mu/\rho = \sigma\,(N_a/A) = N_a/A\,(\phi_{photon} + Z_e\sigma_c + \tau_{pair})$$
$$= \mu/\rho\,(\text{p.E.}) + \mu/\rho\,(\text{C})$$
$$= \sim(\alpha_z Z^3/E^3 + \beta_z f_z(E))$$

- Mass attenuation coefficient is determined by the composition (z) and the quality (energy) of the X-ray beam.
- Linear coefficient is determined by mass atten. coefficient as well as the physical state (mass density) of the medium.

Linear Attenuation Coeff. (cm^{-1})	Mass Attenuation Coeff. (cm^2/g)	Density (g/cm^3)	Thickness of 1 g/cm^3
0.214	0.214	1	water 1 cm
0.196	0.214	0.917	ice 1.09 cm
0.000128	0.214	0.000598	water vapor 1670 cm

Half-value layer (HVL)=0.693/μ

Factors affecting the X-ray beam attenuation:

$$N = N_0 \exp- (\mu_z/\rho)\ \rho\ x$$

- The attenuation is strongly dependent of the effective atomic number of the tissue (muscle= 7.4; fat = 6.6; bone = 12), as well as the x-ray beam quality (energy E).
- Differences in the densities of tissues in the body can produce differences in attenuation. Examples: lung 0.32 g/cm3, lung carcinoma, 1-1.5 g/cm3; fat 0.92 g/cm3, bone 1.85 g/cm3.
- Image formation depends on a differential attenuation between tissues.

The Interaction of Electrons

Energy Loss of Charged Particles (Electrons and Positrons) in the Medium

- **Inelastic collisions with the atomic electron of the material: Energy is transferred from the particle to the atom causing an ionization or excitation of the latter. Statistical in nature, occurring with a certain quantum mechanical probability.**
- ***soft* collisions: only an excitation results;**
- ***hard* collisions: ionization caused.**
- **stopping power: dE/dx, the statistical average of total energy loss pre unit length.**